Mathematics
Kings Worthy Primary School

Tuesday 3rd October 2017

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Hampshire County Council

Continuing Professional Development Standard

National Centre for Excellence in the Teaching of Mathematics

Aims of the session

- To develop an understanding of how calculation strategies are developed in Key Stage 1
- To gain an understanding of age-related expectations in maths at Key Stage 1

When you were at school…..

- How did you feel about maths?
- What were your experiences of maths like?
- How do you feel about maths now?

Maths in school today….

- Maths is fun!
- Children are encouraged to “have a go”!
- They are not afraid to be wrong – mistakes are part of the learning
- Maths is made purposeful and interesting

Why has maths changed?

- Children used to learn "standard methods" – we were shown what to do before understanding was consolidated
- If you couldn't remember, or hadn't understood the methods…you went wrong
- Today, we teach methods that help children to understand the underlying maths and the basic concepts involved
- Children need to develop "number sense" – more insight into mathematics

Today's techniques…

- …are not just about getting the right answer – but about knowing and understanding how you got there
- …are not new – many pre-date the techniques you learned
- …eventually join up with the ones you did in school – but the children understand them thoroughly
- …reduce the chance of mistakes being made
- …build a firm foundation for understanding more complicated mathematics later on

A balanced mathematical diet…

- Number & Place Value
- Addition & Subtraction
- Multiplication & Division
- Fractions
- Measurement
- Geometry

(Position & Direction)

(Properties of shapes)

- Statistics

Year 1expectations

- **Number and place value**
  - Counts to and across 100, forwards and backwards, beginning with 0 or one, or from any given number
- Counts, reads and writes numbers to 100 in numerals; counts in multiples of twos, fives and tens
  - Given a number, identifies one more and one less

- **Addition and subtraction**
  - Represents and uses number bonds and related subtraction facts within 20
    - **Fractions**

Recognises, finds and names a half as one of two equal parts of an object, shape or quantity

Year 2 expectations

- **Number and place value**
- Counts in steps of two, three, and five from 0, and in tens from any number, forward and backward
- Compares and orders numbers from 0 up to 100
- Uses < > and = signs correctly
- Uses place value and number facts to solve problems
- **Addition and subtraction**
- Solves problems with addition and subtraction by:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures; and
- applying an increasing knowledge of mental and written methods.
- Recalls and uses addition and subtraction facts to 20 (fluently) and to 100
- **Multiplication and division**
- Recalls and uses multiplication and division facts for the two, five and 10 multiplication tables, including recognising odd and even numbers.
- Solves problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts
- **Fractions**
- Recognises, finds, names and writes fractions 1/3, ¼, 2/4, and ¾ of a length, shapes and of objects or quantity

Place Value

- Our number system consists of ten digits

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9

- The place of each digit determines its value.

- For example, the “6” digit can represent 6, sixty, six hundred, etc. It depends where we place it…

  6….65….653

Counting

- Counting provides the foundation for calculating.

- Counting is not just reciting '1,2,3,4…' (don't think that your child is too old for counting!)

The Principles of Counting *Gelman and Gallistel (1986)*

Five '*how-to-count*' principles which children must master in order to be able to count reliably:

1.The one-one correspondence principle
2.The stable-order principle
3.The cardinal principle
4.The abstraction principle
5.The order-irrelevance principle

1. The one-one correspondence principle
This involves the assigning of one, and only one, distinct counting word to each of the items to be counted.

2. The stable-order principle
To be able to count also means knowing that the list of words used must be in a repeatable order.

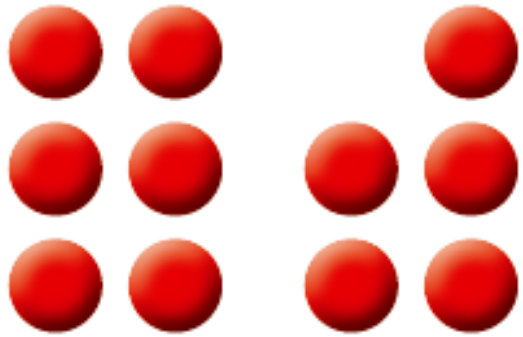
3. The cardinal principle
This principle says that, on condition that the one-one and stable-order principles have been followed, the number name allocated to the final object in a collection represents the number of items in that collection.

4. The abstraction principle
This states that the preceding principles can be applied to any collection of objects, whether tangible or not. To understand this principle, children need to appreciate that they can count non-physical things such as sounds, imaginary objects or even the counting words – as is the case when 'counting on'.

5. The order-irrelevance principle
This principle refers to the knowledge that the order in which items are counted is irrelevant. It does not really matter whether the counting procedure is carried out from left to right, from right to left or from somewhere else, so long as every item in the collection is counted once and only once.

Reasoning: ‘Convince me’

"I know there are an odd number of counters here without counting them all."

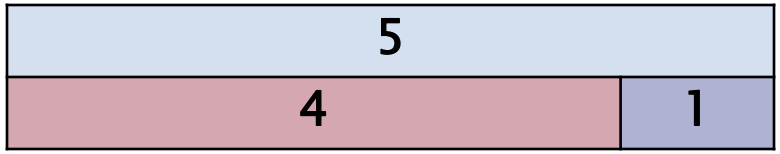


The 'five ness' of 5

Counting to 5 using the five principles

Two numbers that :

- add up to 5 __ + __ = 5
- have a difference of 5
- 5 more than __ is __
- 5 less than __ is __
- bar model



Developing the complexity even if numbers stay the same is a key principle of mastery teaching across the primary curriculum.

(This shows Mastery Teaching in action in Year 1 maths)

Number Lines

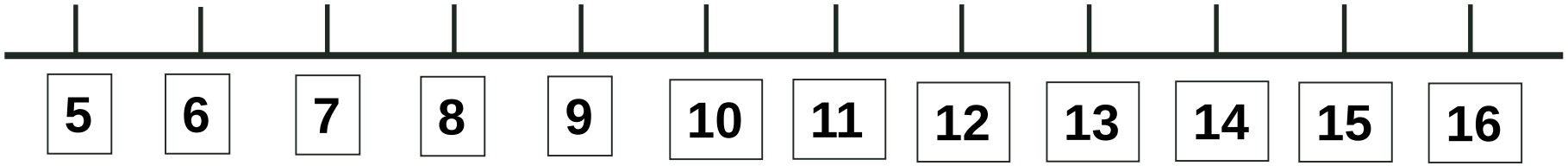
- They help to develop an ability to order numbers, and give children a sense of where numbers sit in our number system
- They allow children to draw a picture – or model – in their heads when calculating
- **Can be used in a range of aspects of mathematics – numbers and the number system, fractions, decimals, percentages, addition, subtraction, multiplication, division, measures, handling data**

The four types of number tracks or number lines

Number tracks

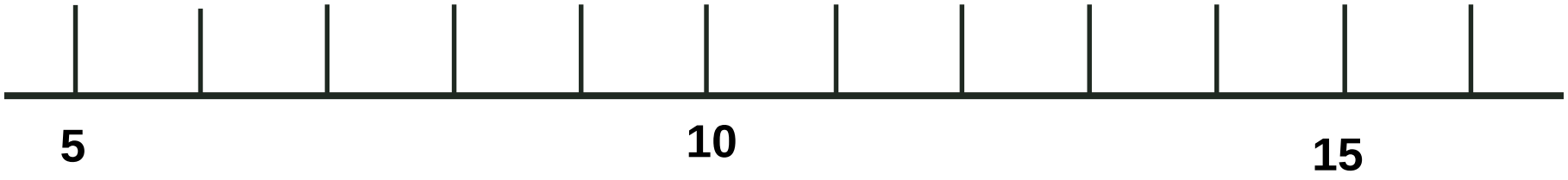
1	2	3	4	5	6	7	8	9	10

Numbered lines



The four types of number tracks or number lines…

- Partly numbered line



- The empty number line

Examples of using a number line

Mrs Stevens put 6 bananas and 5 oranges in the fruit box this morning. How many pieces of fruit are there altogether?

EYFS – Physical objects placed on number line

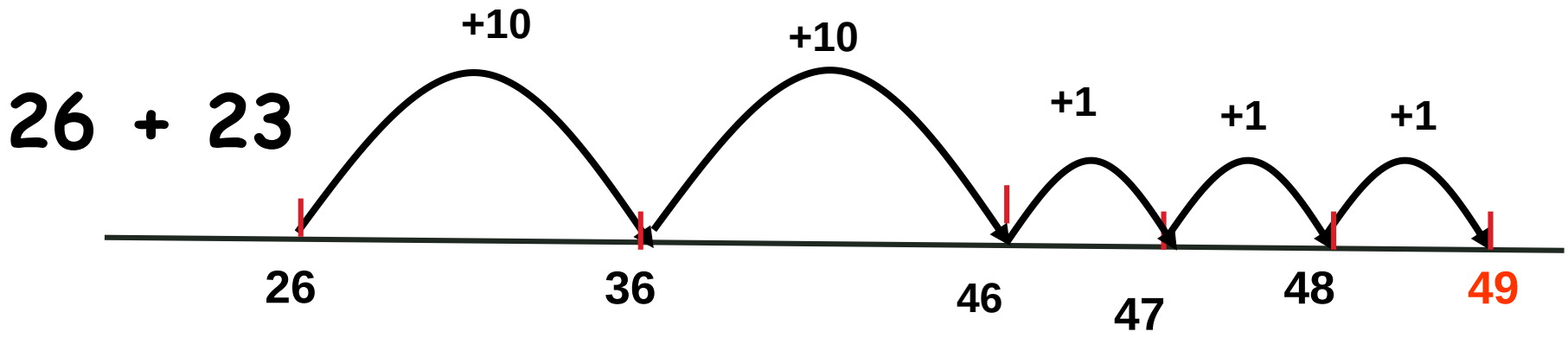
Yr 1 – Find 6 on number line and jump on 5

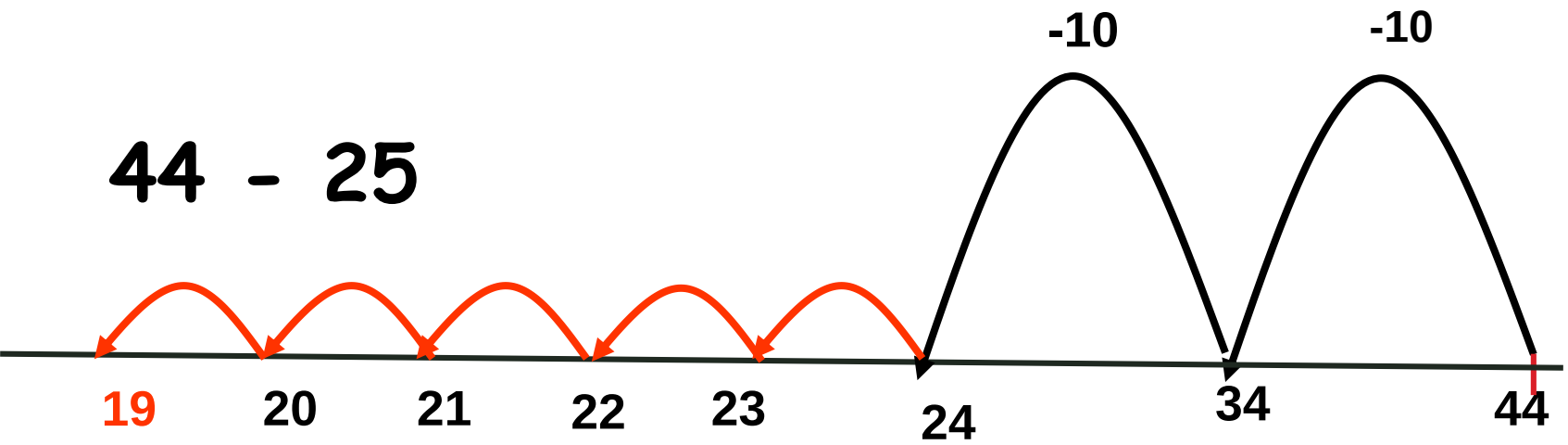
Yr 2 – Finding 6 and knowing it's 4 more to 10 and +1

Partitioning

- This means breaking up numbers into smaller numbers
- All numbers can be partitioned in many different ways
- If children understand about partitioning numbers in different ways, they will calculate more efficiently – and with understanding.

Jump 10's first –
Typical at end of Yr 1/start of Yr 2





Have a go!

- Here are some typical Year 1 and 2 calculations:

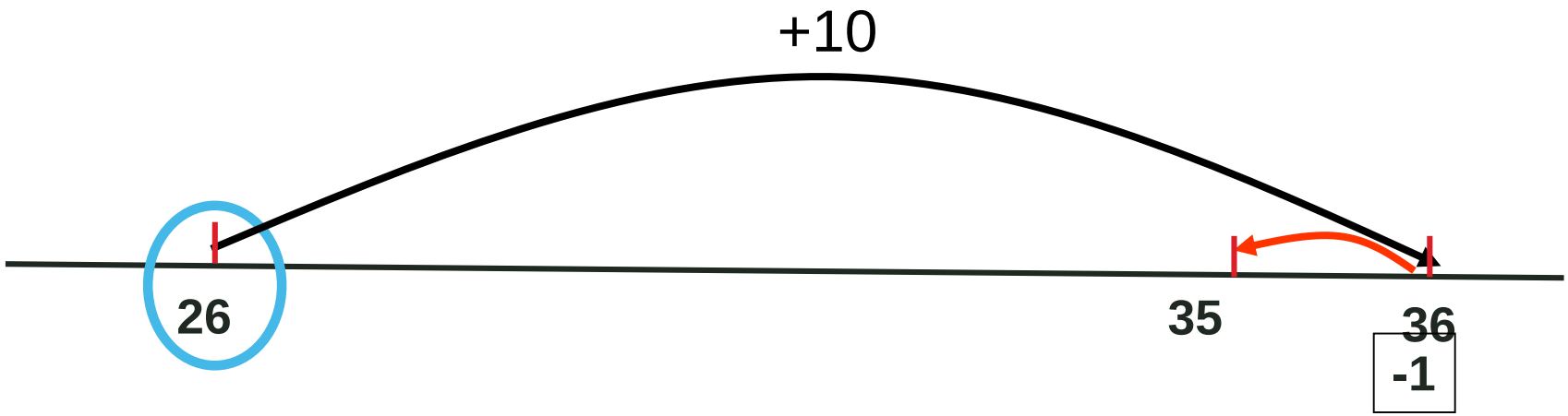
Show your working on an empty number line

There are 16 boys and 15 girls in 2G. How many children altogether?

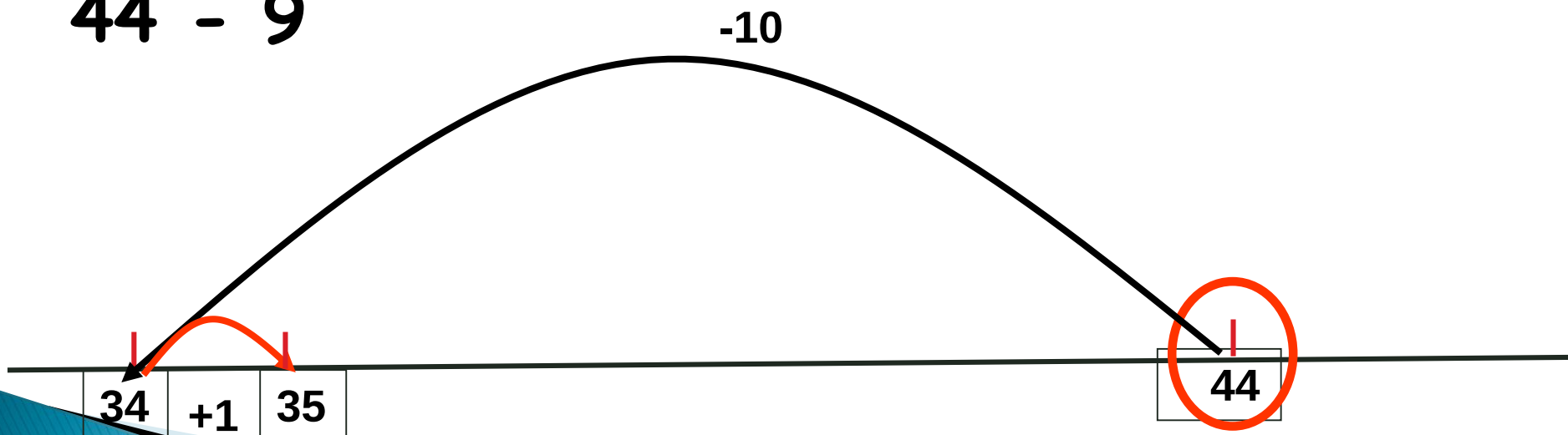
Mr Carr planted 79 trees but the rabbits dug up 43 of them. How many trees were left?

Jump and adjust 26 + 9

"Compensating"



44 - 9



Mental skills and strategies

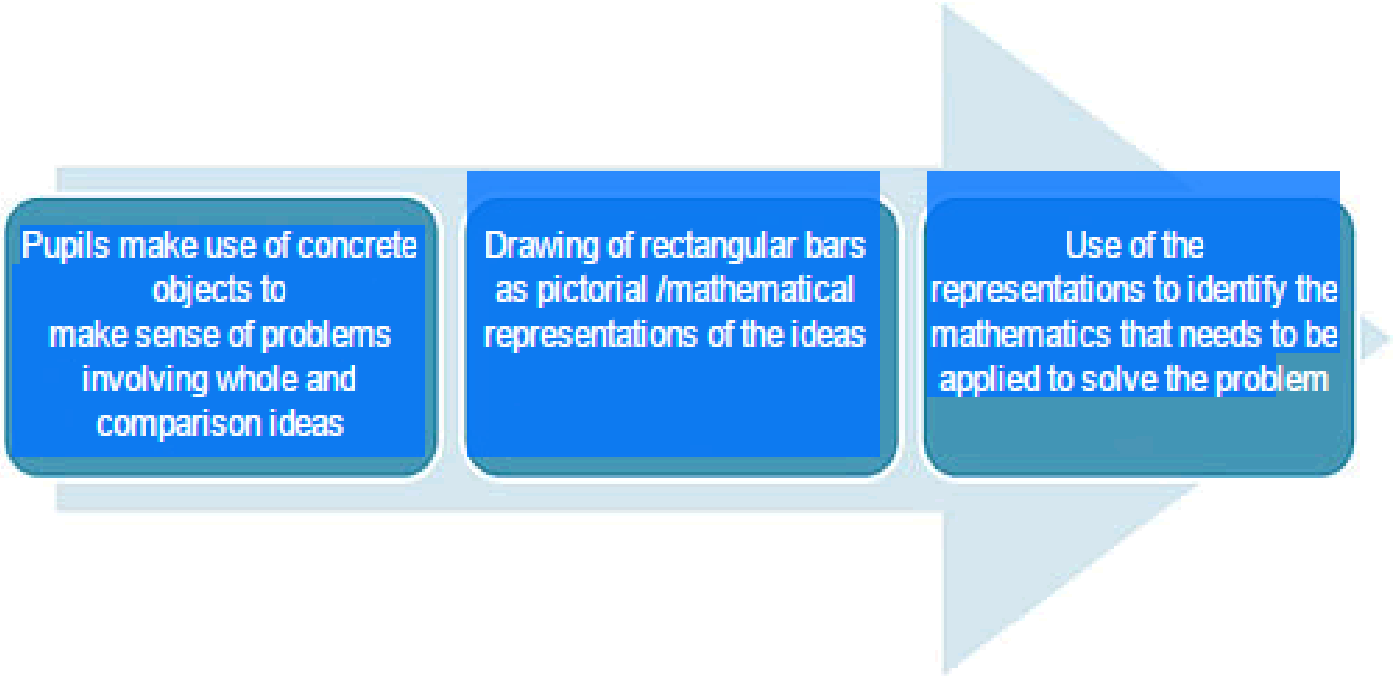
We constantly draw upon **mental strategies** that we know and have internalised when we carry out calculations

- doubling
- adding multiples of ten
- Partitioning
- compensating
- applying known facts

Children need to learn and understand these strategies – and know how and when to apply them.

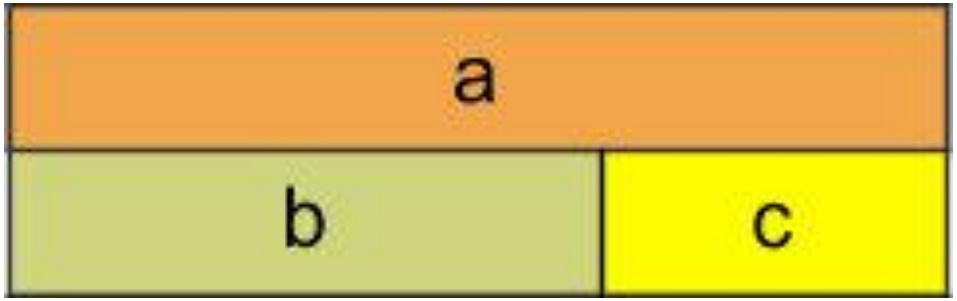
The bar model is used in Singapore and other countries, such as Japan and the USA, to support children in problem solving. It is not a method for solving problems, but a way of revealing the mathematical structure within a problem and gaining insight and clarity as to how to solve it.

It supports the transformation of real life problems into a mathematical form and can bridge the gap between concrete mathematical experiences and abstract representations.



concrete	visual	abstract

Addition and Subtraction



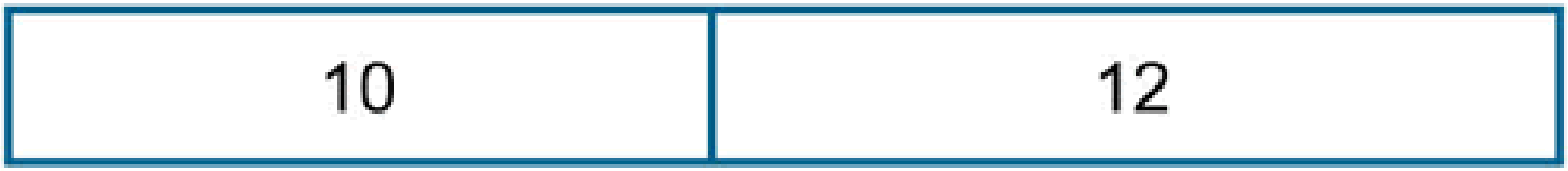
This diagram shows all of the following relationships;
a = b + c ; a = c + b ; a – b = c ; a – c = b

To prepare young children for the bar model it is a good idea to encourage them to line up objects in a linear arrangement when representing addition and subtraction problems.

Such arrangements will also help children to organise their counting. The physical objects can then be replaced, in time, with linking cubes and with a bar drawn next to it. The question can then be asked “what’s the same, what’s different?” to support the children in their reasoning and in making sense of the bar as an abstract representation of the physical objects.

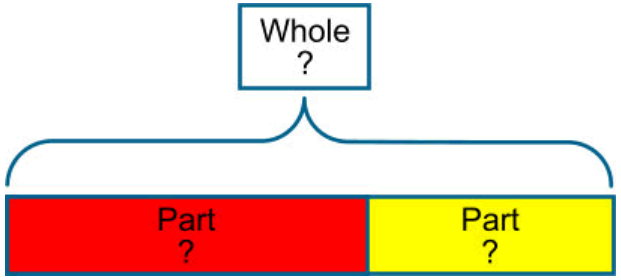
It is useful for children to work in pairs with one manipulating the cubes, while the other records by drawing the bars and then writing the number sentence underneath. The children can then swap roles.

Sam had 10 red marbles and 12 blue marbles. How many marbles did he have altogether?



$10 + 12 = 22$

In problems involving addition and subtraction there are three possible unknowns as illustrated below and given the value of two of them the third can be found.



Practical objects CONCRETE

Bar model to represent objects VISUAL

Calculation strategies ABSTRACT

Why are images important?

- Resources, models and images help children to visualise and understand mathematical concepts
- They build up – and remember - the mental picture in their minds
- They should be available throughout the primary years. Children will rely on them less and less

Concrete → Visual → Abstract

Practical objects	Models and Images	Informal and formal calculation strategies

9	
5	4

Related facts:
5+ 4 = 9
4 + 5 = 9
9- 5 = 4
9 – 4 =5

Word problems:
Tom and Sam have 9 apples. If Tom has 4 apples, how many apples does Sam have?

What is 1 more or one less than 9?

9 is 1 less than __
__ is 1 more than 9

Use the first number sentence to complete the second number sentence.

4 + 3 = ☐	7 + ☐ = 9
7 – ☐ = 4	9 – ☐ = 7

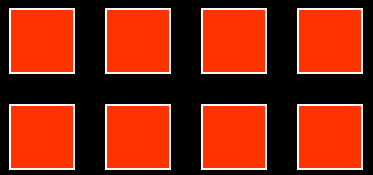
5 + 2 = ☐	☐ + 3 = 9
☐ – ☐ = 2	☐ – ☐ = ☐

Jill thought of a number. Ten more than her number is 19. What was the number she was thinking of?

Multiplication in KS1

Arrays are important because they provide a good visual image of the multiplication that links closely to the concept of repeated addition.

2 + 2 + 2 + 2



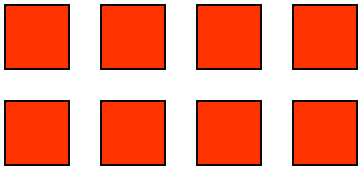
4 x 2 = 8

2 x 4 = 8

They are the visual image which leads directly into grid multiplication a key strategy in KS2

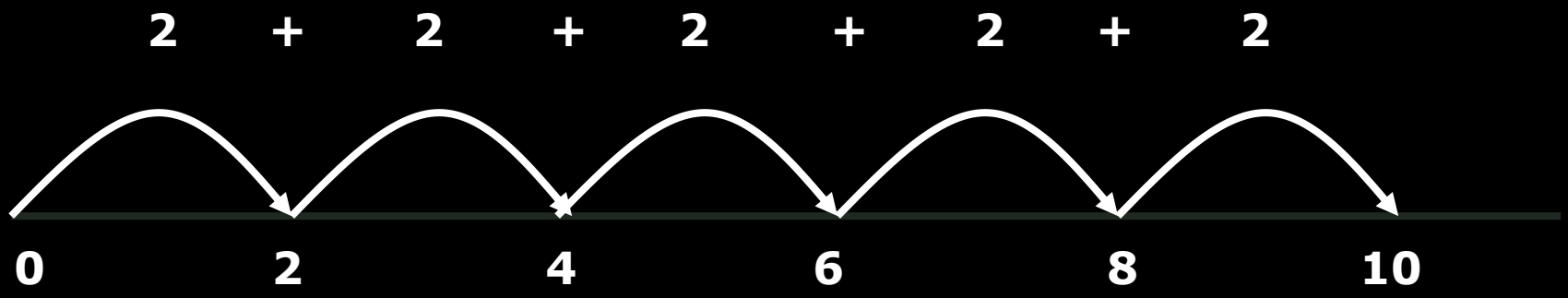
Arrays

- They show the link between multiplication and division – these are not concepts taught in isolation



2 x 4 = 8
4 x 2 = 8
8 ÷ 2 = 4
8 ÷ 4 = 2

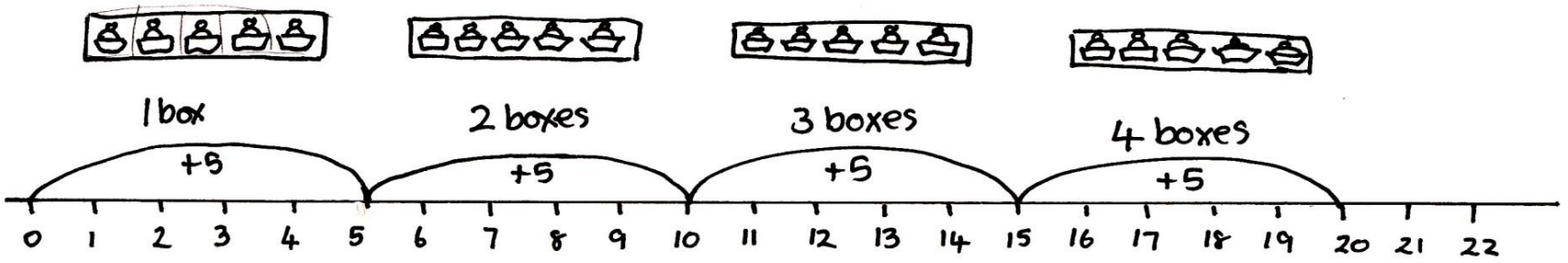
Number lines for multiplication



This image can be expressed as 2 multiplied by 5, two, five times, 5 groups of 2, 5 lots of 2 and 5 hops of 2 on a number line.

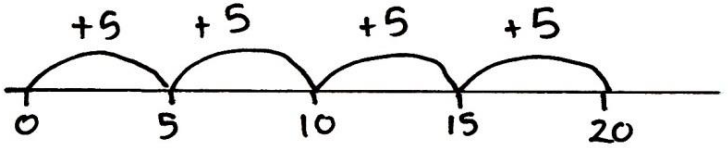
Multiplication on a number line

There are 5 cakes in one box. How many cakes in 4 boxes?



$5 \times 4 = 20$

5 cakes in each box. 4 boxes equals 20 cakes altogether.



$5 \times 4 = 20$

Division in Key Stage 1

Division can be GROUPING or SHARING

Sharing:
We have 20 marbles in the box to share between 4 children. How many marbles do they get each?
=5 marbles

Grouping:
We have 20 marbles and they come in packets of 4. How many packets were there?
=5 packets

KS1 strategies for division

- I have 12 lollipops and I want to put 4 lollipops in each bag. How many bags will I need?
- I have 12 lollipops and I share them among 4 people. How many lollipops will they have each?

When tackling a mathematical problem, children will ask themselves…

- What do I know about these numbers?
- Can I do this in my head?
- Do I know the approximate size of the answer?
- If I can't do it all in my head, what do I need to write down to help me?

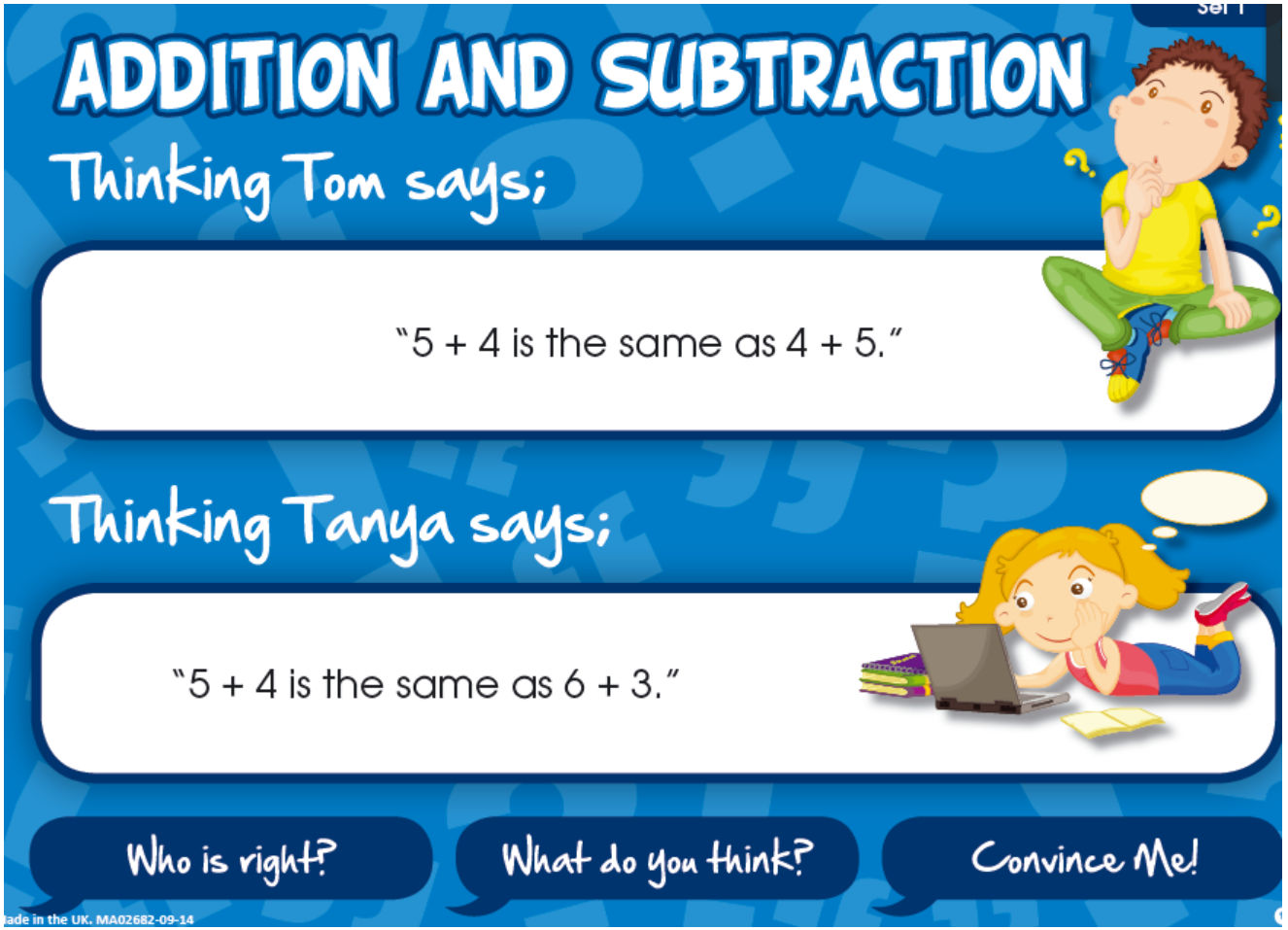
Year 1 Problems

1. Ebony has 5p and Daniel has 8p. How much do they have altogether?
2. A lolly costs 6p. Amrit paid with a 10p coin. How much change does he get?
3. Michael says that 16 + 5 = 21. Is he correct?
4. I think of a number. I subtract 5. The answer is 4. What was my number?
5. How many gloves are there altogether in 6 pairs of gloves?
6. Twelve people are split into two groups. How many are in each group?
7. Mrs Morton puts five 5p coins into her purse. How much is in her purse altogether?

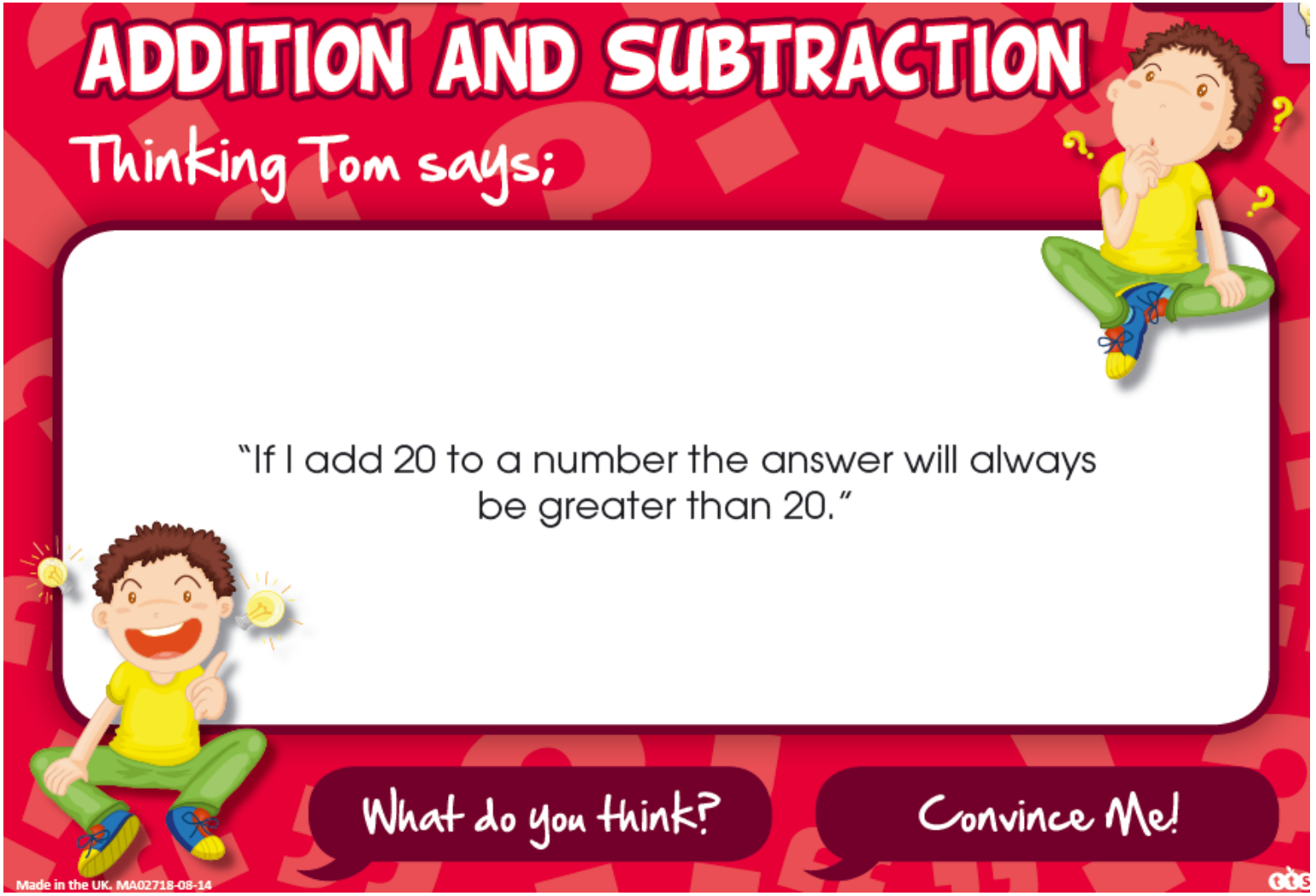
Year 2 Problems

1. Dylan has 37 coloured pencils and he buys 30 more. How many does he have now?
2. Janie has 40 beads. She loses 25 of them. How many does she have left?
3. What is the difference between seventy six and thirty five?
4. I think of a number. I subtract 5. The answer is 4. What was my number?
5. Last week Ellie got £1.00 pocket money. She spent half of it. How much has she got left?
6. A tub contains 24 coins. Saj takes 5 coins. Joss takes 10 coins. How many coins are left in the tub?
7. Amelia writes the calculation below as a multiplication calculation? What might she write?
3 + 3 + 3 + 3 + 3 = 15
8. Mr Siddique shares £18 equally between his three sons. How much does each son get?
9. Charlotte-May had to find a 1/4 of a number. Her answer was 4. What number did she start with?
10. Danny cuts his pizza into 8 equal slices. He eats 3/4 of the pizza and gives the rest to his dog, Gruff. How many pieces does Danny eat?

Children need to have the confidence and resilience to explore ideas in maths…



Mastery with Greater Depth



Thinking outside of the box!

Why does this work? ***Convince me.***

Use reasoning prompts to do this:

- Why do you think that …?
- Can you explain why that is right?
- How do you know?
- How did you reach that conclusion?
- What might explain that …?
- How is that possible?
- Can you show me …?
- Is there another way …?
- What explanation do you think is best …?
- Have you tried all the possible cases?
- Does it always work? Why?
- What do you notice when …?

A useful checklist for what to look out for when assessing a pupil's understanding might be:

A pupil really understands a mathematical concept, idea or technique if he or she can:

- **describe** it in his or her own words;
- **represent** it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)
- **explain** it to someone else;
- **make up his or her own examples** (and non-examples) of it;
- **see connections** between it and other facts or ideas;
- **recognise it in new situations and contexts**;
- **make use of it in various ways**, including in new situations.

"Above all – have fun! Children often say that maths is their favourite subject and they get quicker and better at it when they understand what they are doing. There's no need for you to "teach" your child: you help by helping them to explain their thinking and their understanding"

From "Count on me – 200 ways to help with mathematics"

- “It’s the school’s job to provide the structured learning….Your role is to nurture and support your child’s mathematical knowledge away from school, to bring it into their real lives and, most important of all, to turn it into an exciting adventure”

From “Maths for Mums and Dads” – Rob Eastaway and Mike Askew

Have your questions been answered?