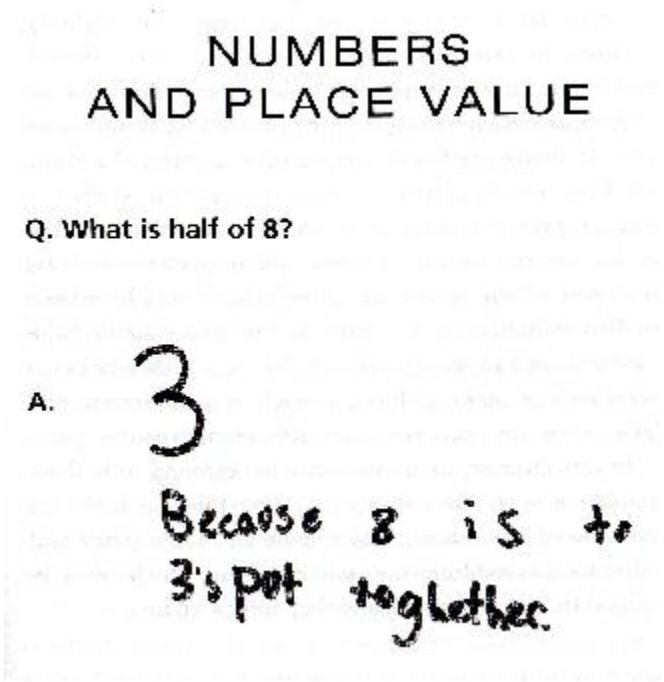

NUMBERS
AND PLACE VALUE

Q. What is half of 8?

A. 3

Because 8 is to 3's put toglether.

Mathematics

Kings Worthy Primary School

Tuesday 10th October 2017

Miss Evers

Maths Manager

Hampshire County Council

Continuing Professional Development Standard

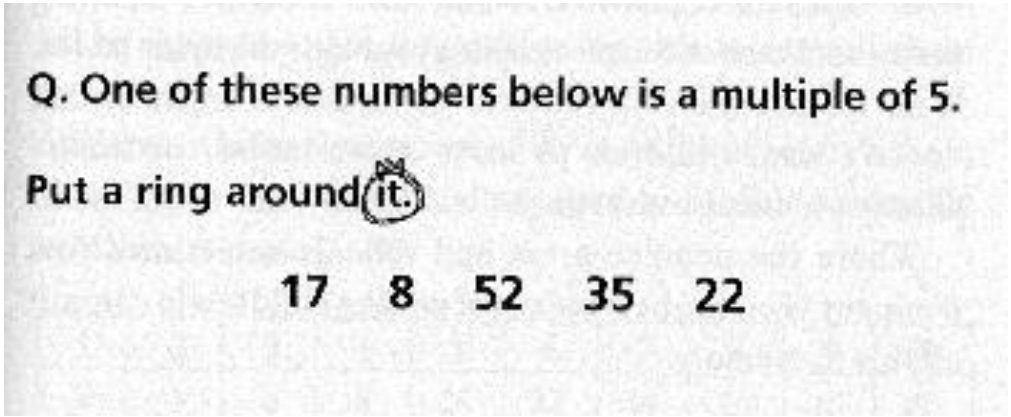
National Centre for Excellence in the Teaching of Mathematics

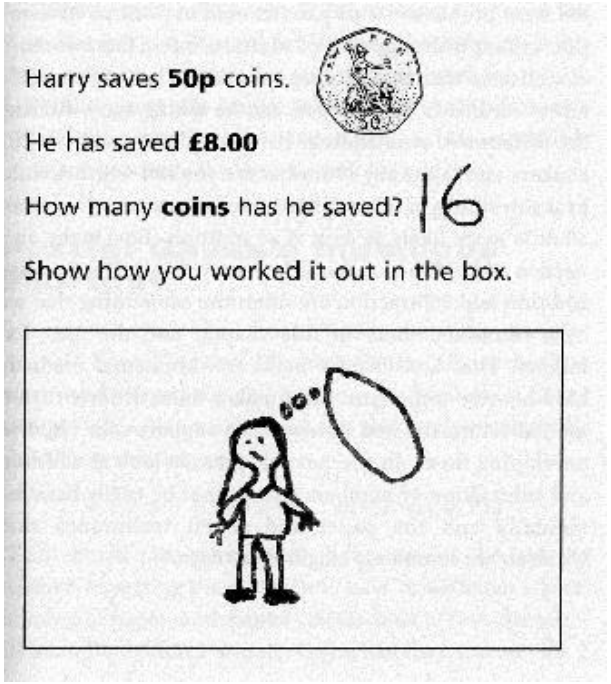
Aims of the session

- To develop an understanding of how calculation strategies are developed from Key Stage 1 to Key Stage 2
- To gain an understanding of age-related expectations in maths at Key Stage 2, with a focus on Year 3 and 4.

When you were at school…

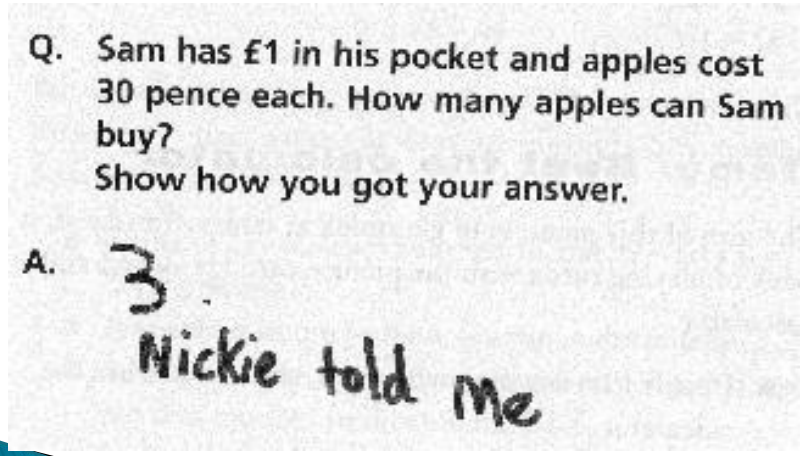
- How did you feel about maths?
- What were your experiences of maths like?
- How do you feel about maths now?

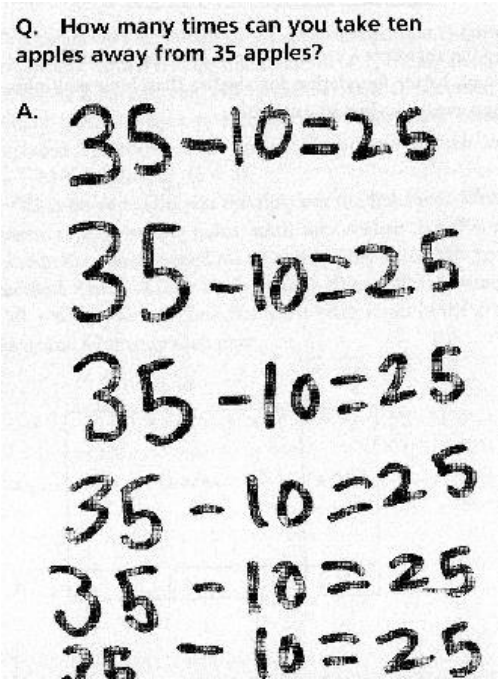




Maths in school today…

- Maths is fun!
- Children are encouraged to "have a go"!
- They are not afraid to be wrong – mistakes are part of the learning
- Maths is made purposeful and interesting





Why has maths changed?

- Children used to learn "standard methods" – we were shown what to do before understanding was consolidated
- If you couldn't remember, or hadn't understood the methods…you went wrong
- Today, we teach methods that help children to understand the underlying maths and the basic concepts involved
- Children need to develop "number sense" – more insight into mathematics

Today's techniques…

- …are not just about getting the right answer – but about knowing and understanding how you got there
- …are not new – many pre-date the techniques you learned
- …eventually join up with the ones you did in school – but the children understand them thoroughly
- …reduce the chance of mistakes being made
- …build a firm foundation for understanding more complicated mathematics later on

A balanced mathematical diet…

- Number and Place Value
- Addition & Subtraction
- Multiplication & Division
- Fractions
- Measurement
- Geometry

(Position & Direction)

(Properties of shapes)

- Statistics

Year 3

- Adding tens or hundreds to 3-digit numbers
- Formal written methods for addition/subtraction
- Counting in tenths
- Comparing, ordering, adding & subtracting fractions with common denominators
- Identifying angles larger than/smaller than right angles
- Identify horizontal, vertical, parallel and perpendicular lines
- Tell time to the nearest minute, including 24-hour clock and using Roman numerals
- Know the number of seconds in a minute and the number of days in each month, year and leap year

Year 4

- Solving problems with fractions and decimals to two decimal places
- Rounding decimals to whole numbers
- Roman numerals to 100
- Recognising equivalent fractions
- Knowing equivalent decimals to common fractions
- Dividing by 10 and 100 (incl. with decimal answers)
- Using factor pairs
- Translation of shapes
- Finding perimeter/area of compound shapes
- Solve time conversion problems

Place Value

- Our number system consists of ten digits

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9

- The place of each digit determines its value.

- For example, the “6” digit can represent for 6, sixty, six hundred. It depends where we place it…

  6….65….653

Counting

- Counting provides the foundation for calculating.

- Counting is not just reciting '1,2,3,4…' (don't think that your child is too old for counting!)

The Principles of Counting *Gelman and Gallistel (1986)*

Five '*how-to-count*' principles which children must master in order to be able to count reliably:

1.The one-one correspondence principle
2.The stable-order principle
3.The cardinal principle
4.The abstraction principle
5.The order-irrelevance principle

1. The one-one correspondence principle
This involves the assigning of one, and only one, distinct counting word to each of the items to be counted.

2. The stable-order principle
To be able to count also means knowing that the list of words used must be in a repeatable order.

3. The cardinal principle
This principle says that, on condition that the one-one and stable-order principles have been followed, the number name allocated to the final object in a collection represents the number of items in that collection.

4. The abstraction principle
This states that the preceding principles can be applied to any collection of objects, whether tangible or not. To understand this principle, children need to appreciate that they can count non-physical things such as sounds, imaginary objects or even the counting words – as is the case when 'counting on'.

5. The order-irrelevance principle
This principle refers to the knowledge that the order in which items are counted is irrelevant. It does not really matter whether the counting procedure is carried out from left to right, from right to left or from somewhere else, so long as every item in the collection is counted once and only once.

Number Lines

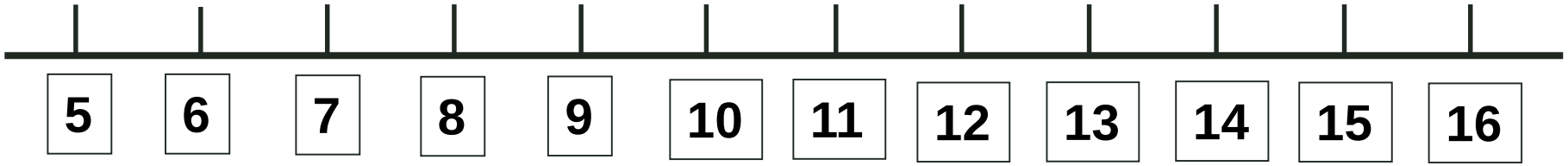
- They help to develop an ability to order numbers, and give children a sense of where numbers sit in our number system
- They allow children to draw a picture – or model – in their heads when calculating
- **Can be used in a range of aspects of mathematics – numbers and the number system, fractions, decimals, percentages, addition, subtraction, multiplication, division, measures, handling data**

The four types of number tracks or number lines

Number tracks

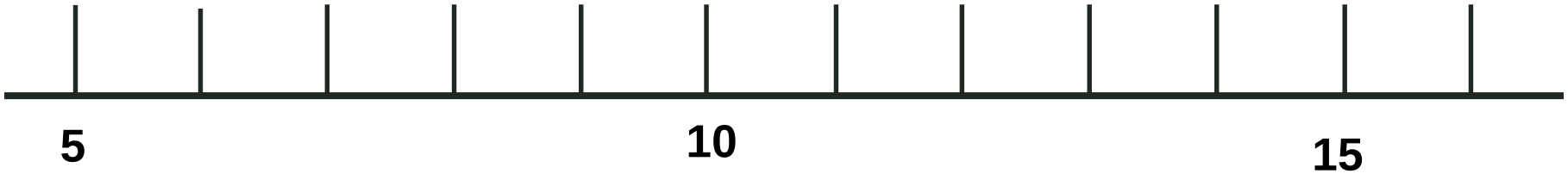
1	2	3	4	5	6	7	8	9	10

Numbered lines



The four types of number tracks or number lines…

- Partly numbered line

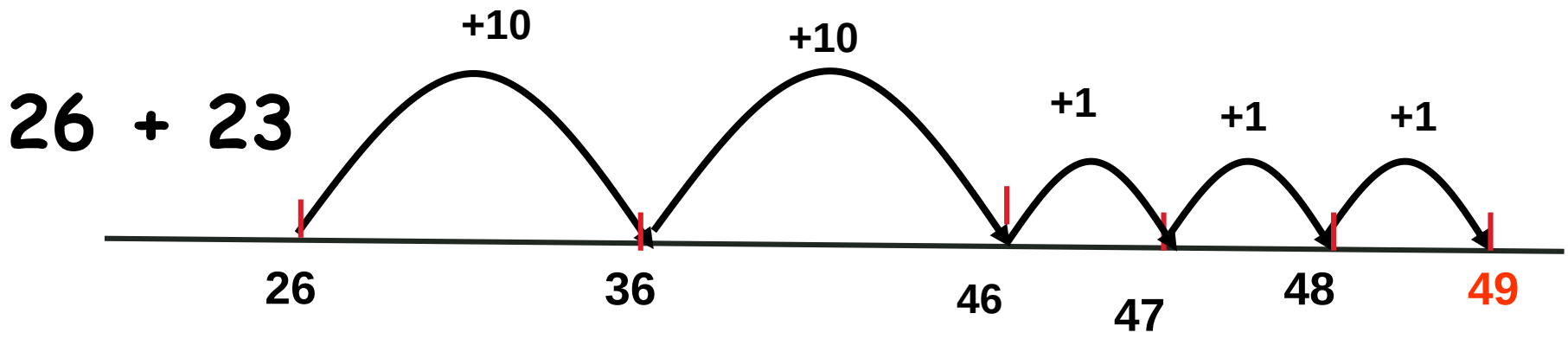


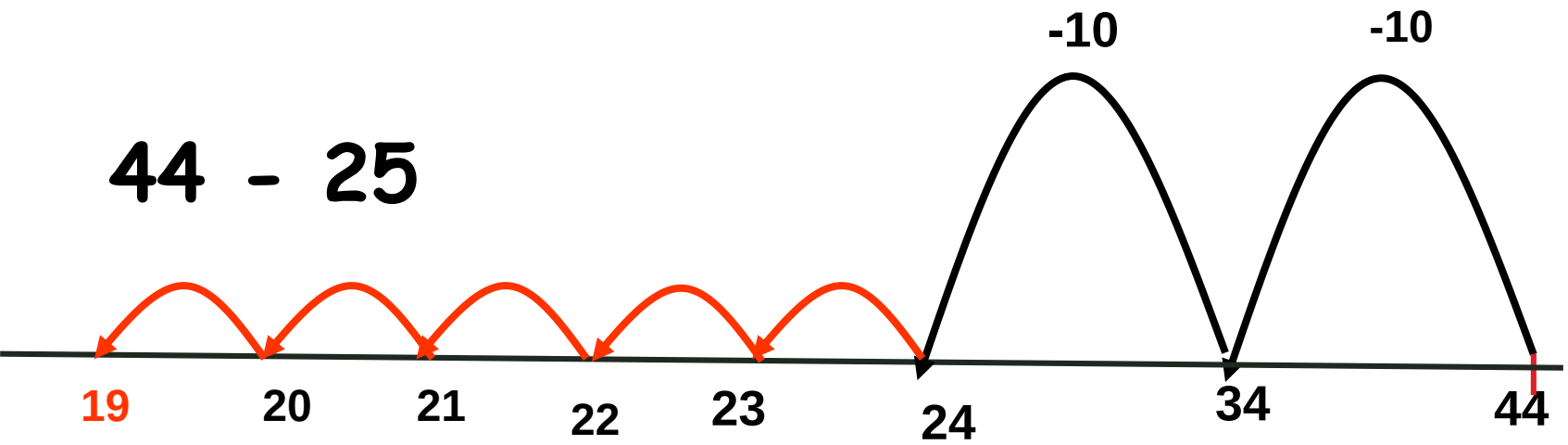
- The empty number line

Partitioning

- This means breaking up numbers into smaller numbers
- All numbers can be partitioned in many different ways
- If children understand about partitioning numbers in different ways, they will calculate more **efficiently** – and with understanding.

Jump 10's first –
Typical at end of Yr 1/start of Yr 2





Mental skills and strategies

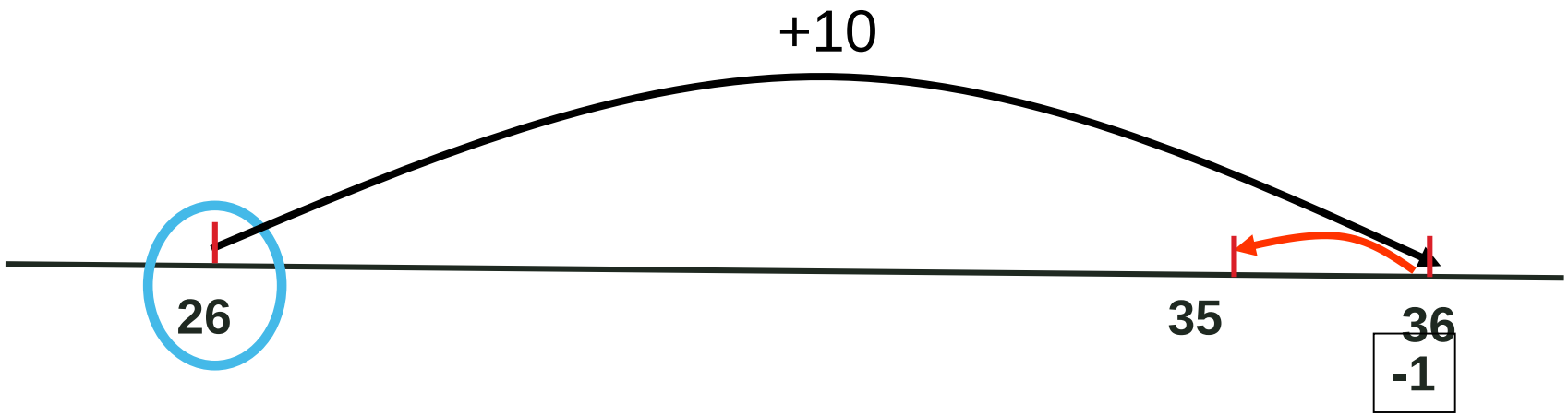
We constantly draw upon **mental strategies** that we know and have internalized when we carry out calculations

- doubling
- adding multiples of ten
- partitioning
- compensating
- applying known facts

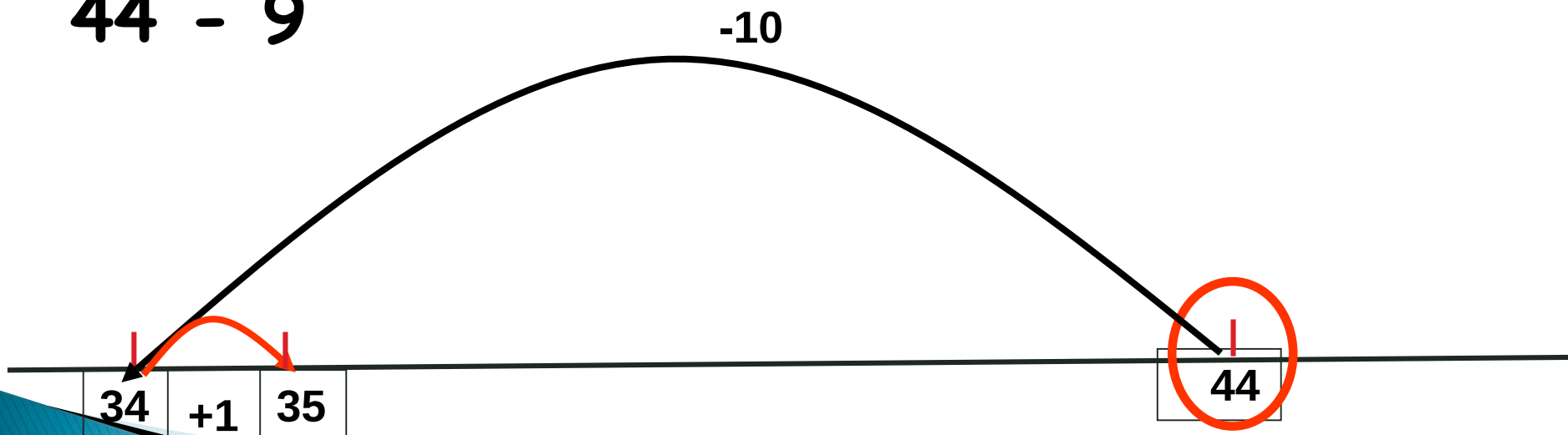
Children need to learn and understand these strategies – and know how and when to apply them.

Jump and adjust 26 + 9

"Compensating"



44 - 9



Example of calculations:

- Here are some typical Year 3 and Year 4 calculations:

There are 247 boys and 354 girls in a school. How many children altogether?

Mr Carr planted 687 seeds but the rabbits dug up 298 of them. How many seeds were left?

Year 3 Problems

1. There are 334 children at Springfield School and 75 at Holy Trinity Nursery. How many children are there altogether?
2. Gemma collected 293 badges but she gave 45 of them to her friend, Rebecca. How many badges did she have left?
3. Aiden has seven marbles and Harvey has fifteen. They decide to share them equally between them. How many do they get each?
4. Seven people each put five pens into a pot. Carmen then takes out fifteen pens. How many pens are left?
5. If you spend 61p at the corner shop, how much change do you get from £1.00?
6. If five apples cost fifty pence, how much would two apples cost?
7. Emma buys seven markers for 30p each. How much change does she get from £3.00?
8. A bookcase in the library holds 5 shelves with 46 books on each shelf. How many books are there in the bookcase altogether?
9. How many 5p stickers can Alexis buy with his 55p pocket money?
10. Which is the larger amount, one third of £60 or one quarter of £88?
11. A computer game is £24 in the sale. This is one quarter off its original price. How much did it cost before the sale?

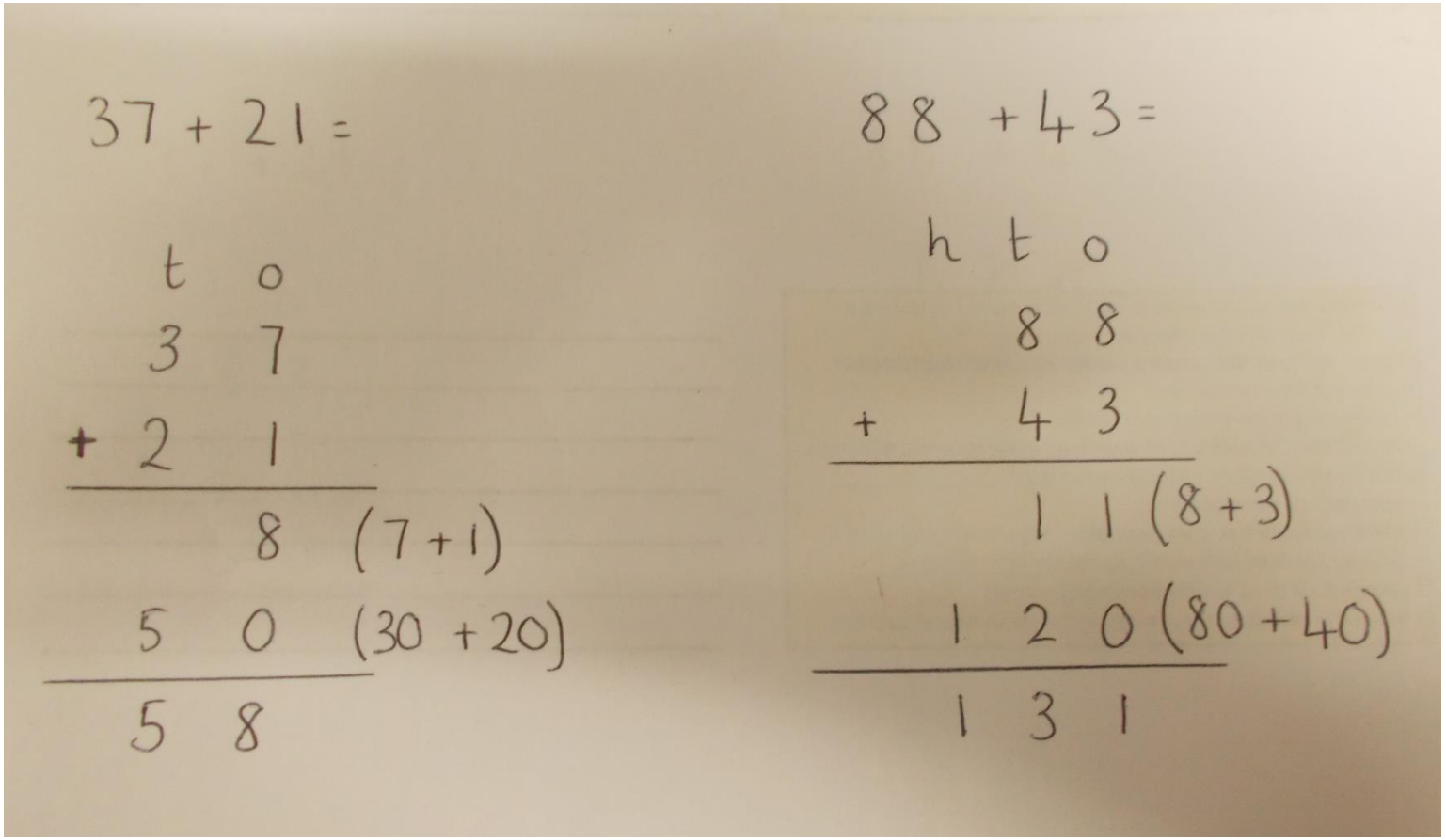
Year 4 Problems

1. Martin has saved £6.78 and spends £4.69. How much does he have left?
2. Sally has 40 football cards. She gives 25 of them away. How many does she give away?
3. Sally has 30 football cards. She gives 25 of them to her friend. How many does she have left?
4. 8 children each download 59 songs to play on their iPod. How many songs do they have altogether?
5. Calculate how many fives there are in 85?
6. At the dressmakers, Debbie buys buttons weighing 3 grams each. If she has 81 grams of buttons, how many buttons does she buy?
7. Kelly buys four fifths of the shop's oranges. If the shop had 20 oranges, how many does she have?

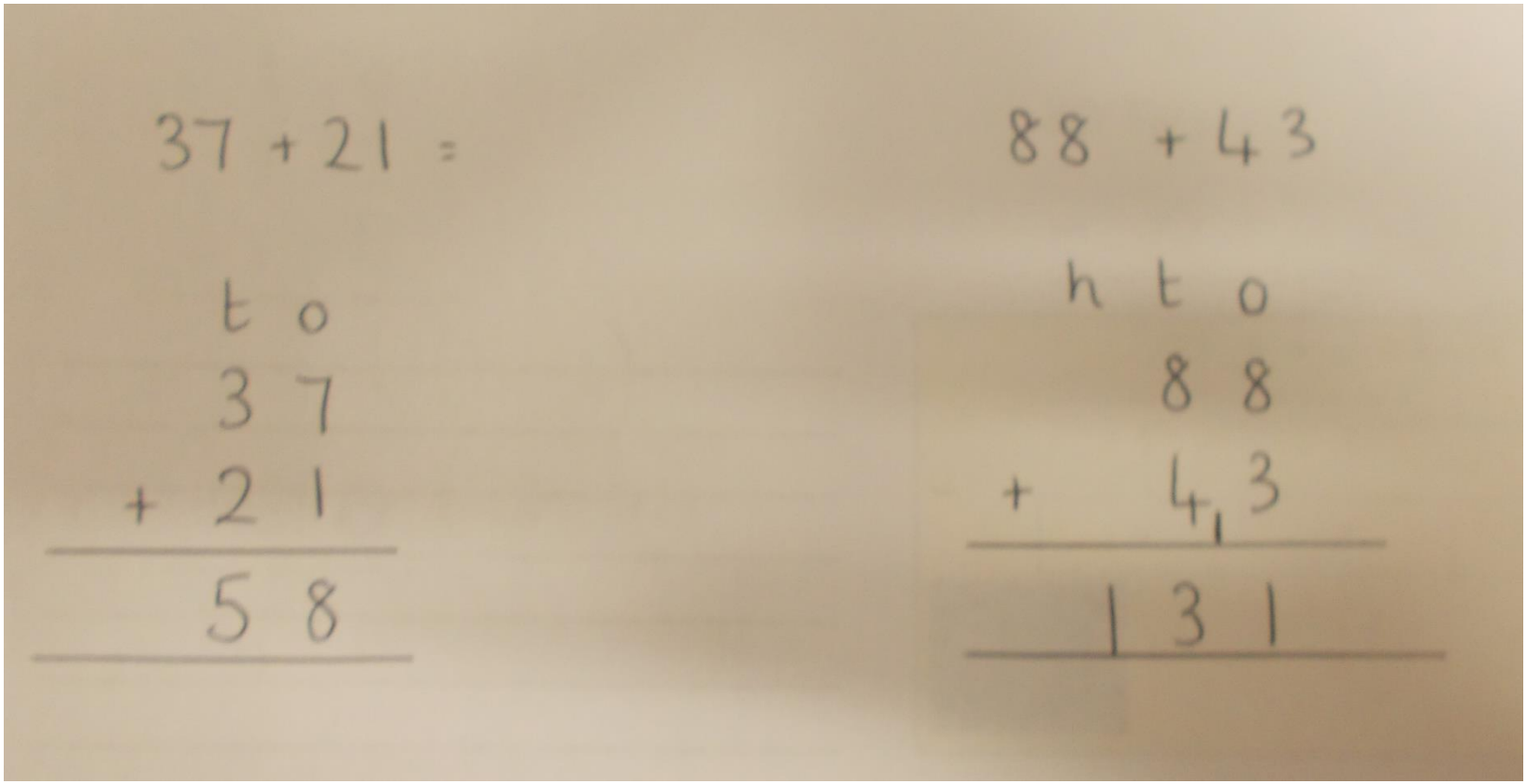
Progression in addition methods

- Numberlines (see previous slides)
- Column (expanded)
- Column
- Formal compact method (see KS2 parents presentation)

Addition – column expanded



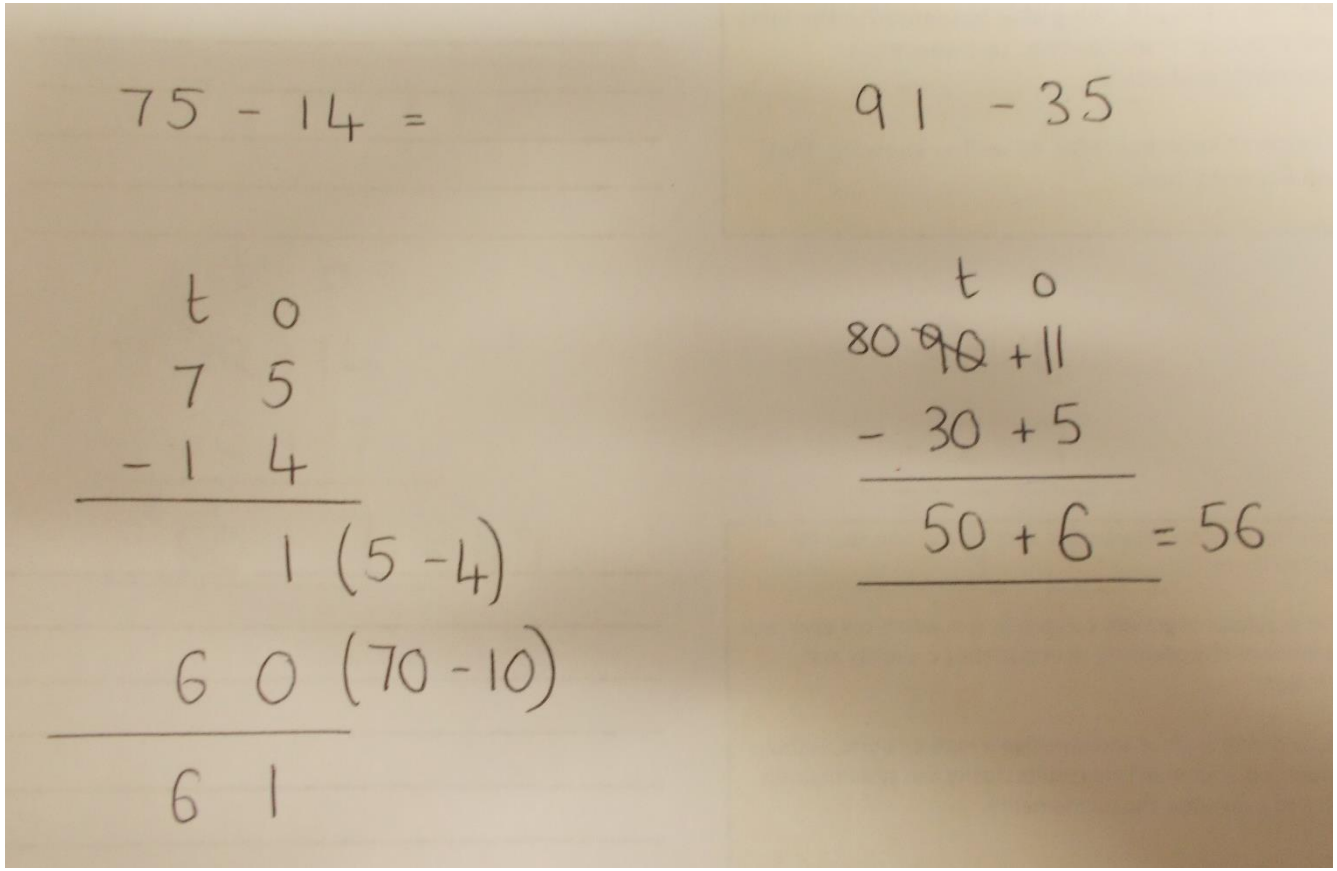
Addition – column method



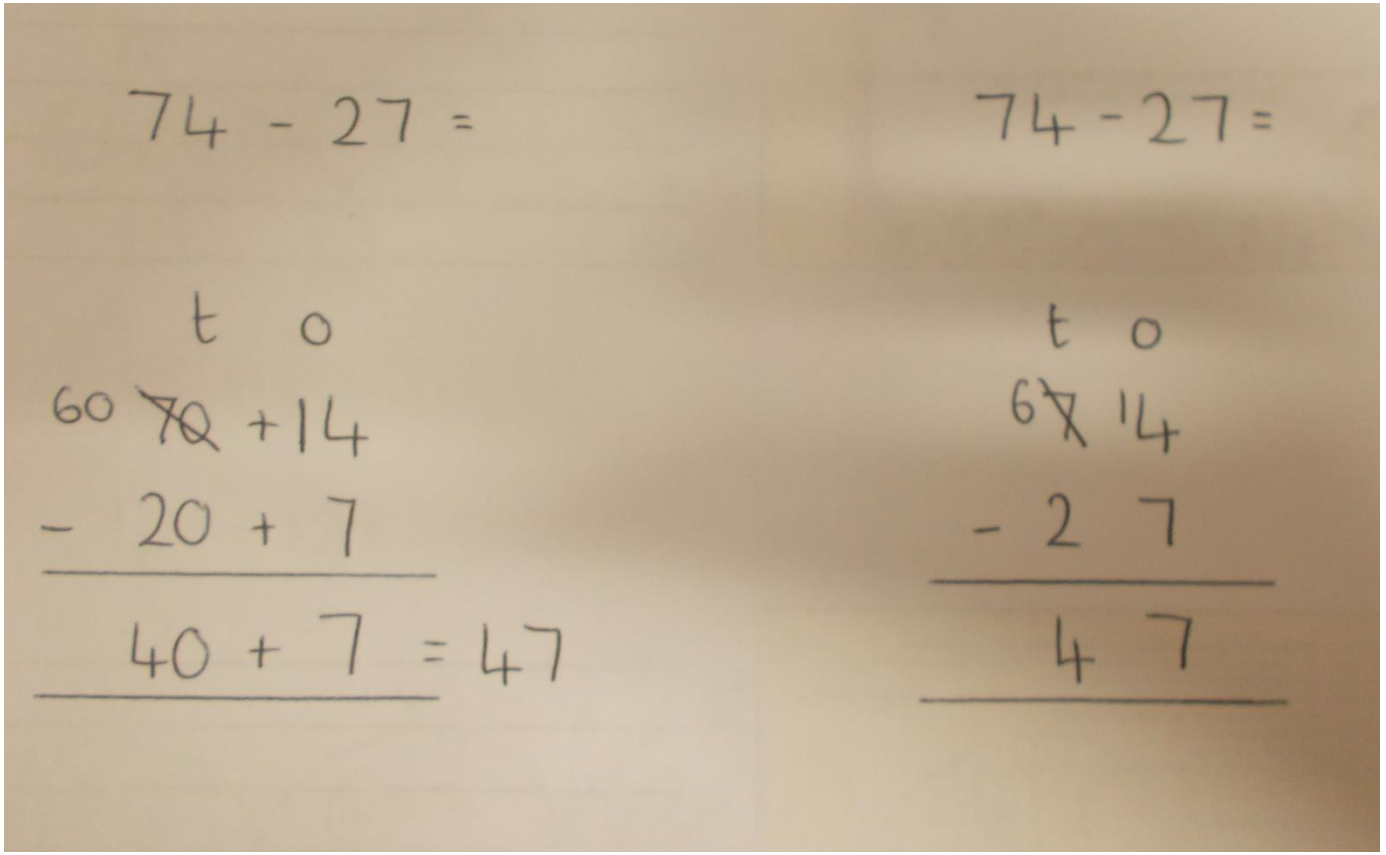
Progression in subtraction methods

- Numberlines (see previous slides)
- Column method (expanded)
- Column method
- Formal compact method (see KS2 presentation)

Subtraction – column expanded



Subtraction – column expanded



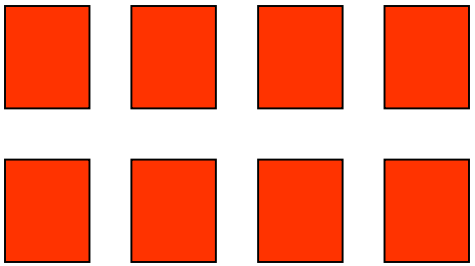
Progression in multiplication strategies

- ‘Sets of’ using objects
- Arrays
- Repeated addition on a number line
- Grid (2digit x 1digit)
- Grid (2digit x 2digit)
- Expanded short
- Formal compact method (see Upper KS2 presentation)

Multiplication in KS1

Arrays are important because they provide a good visual image of the multiplication that links closely to the concept of repeated addition.

2 + 2 + 2 + 2



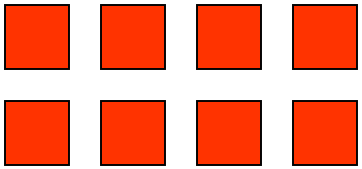
4 x 2 = 8

2 x 4 = 8

They are the visual image which leads directly into grid multiplication – a key strategy in KS2

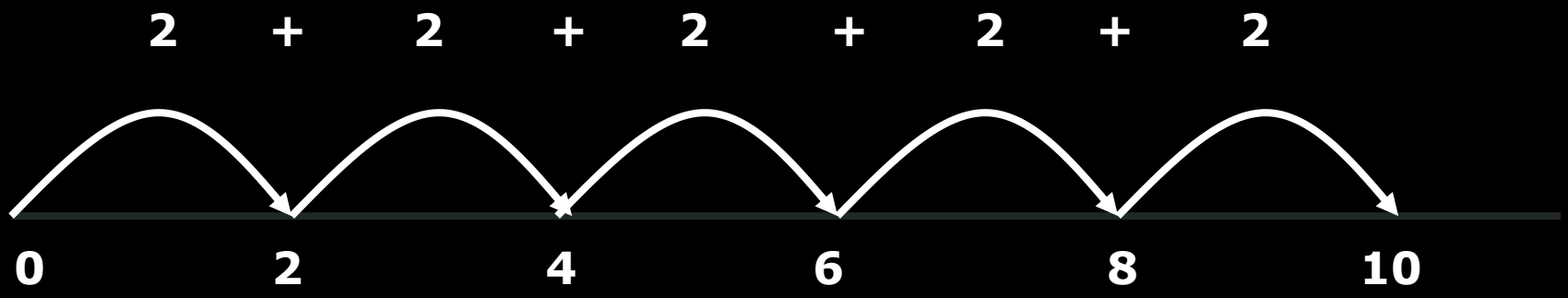
Arrays

- They show the link between multiplication and division – these are not concepts taught in isolation



2 x 4 = 8
4 x 2 = 8
8 ÷ 2 = 4
8 ÷ 4 = 2

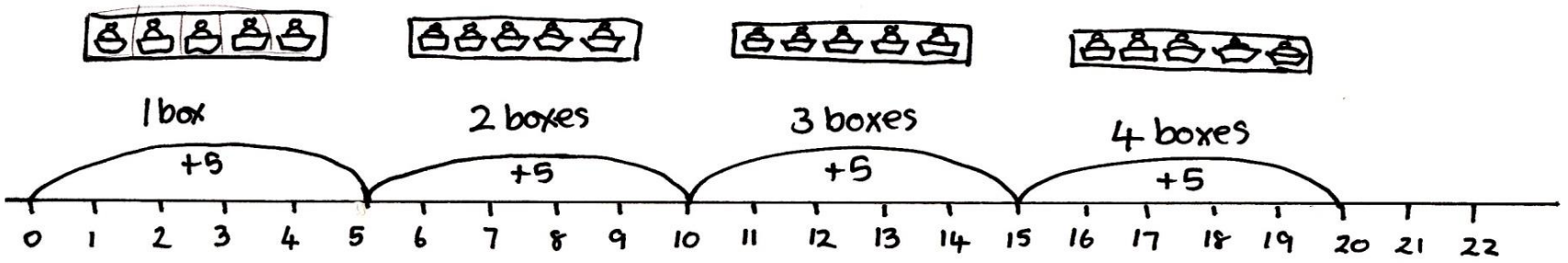
Number lines for multiplication



This image can be expressed as 2 multiplied by 5, two, five times, 5 groups of 2, 5 lots of 2 and 5 hops of 2 on a number line.

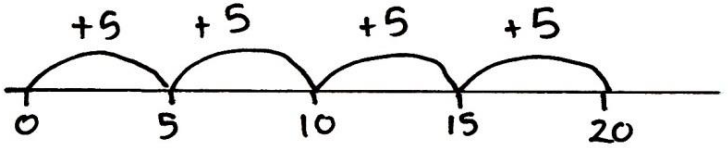
Multiplication on a number line

There are 5 cakes in one box. How many cakes in 4 boxes?



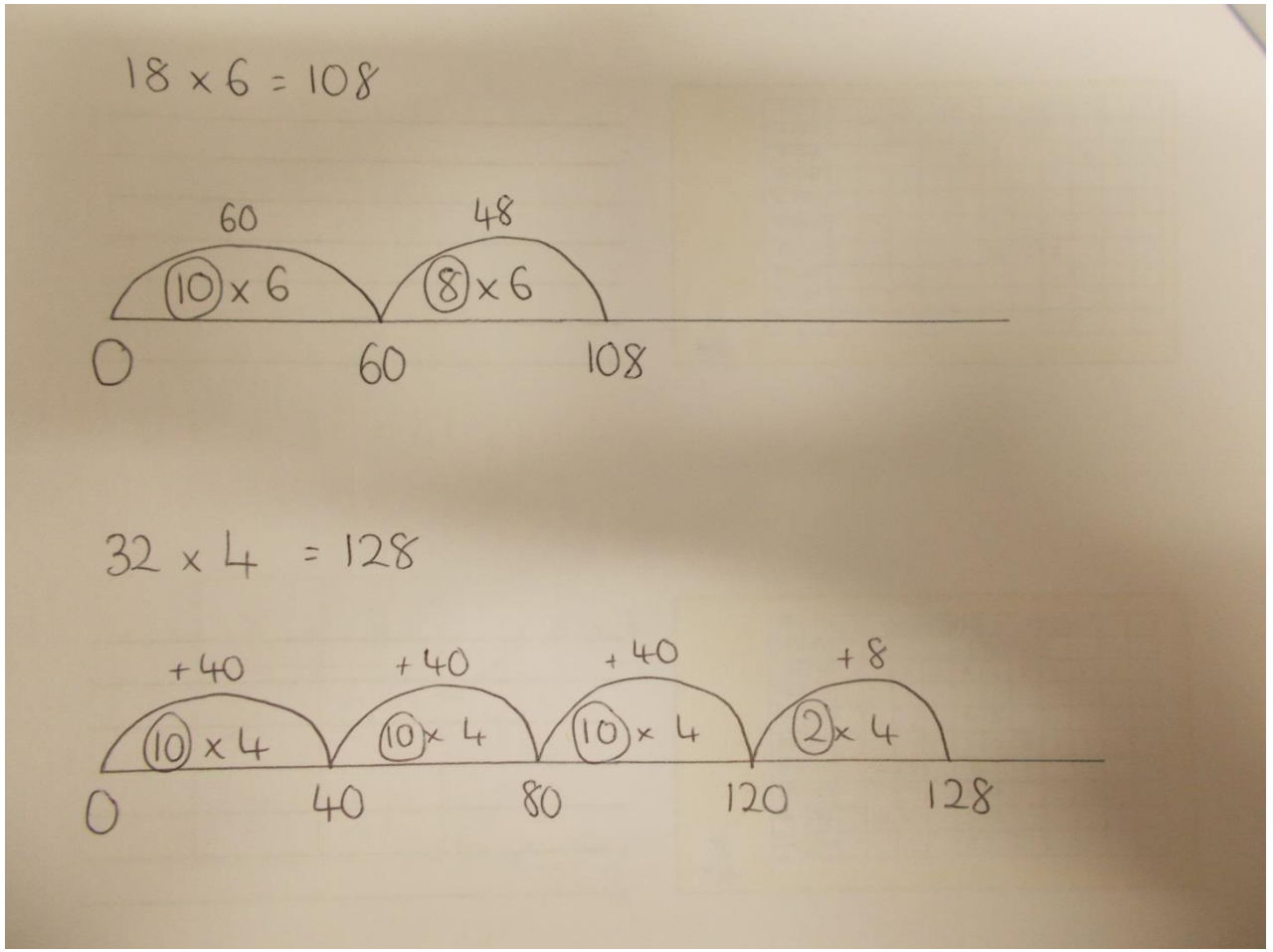
$5 \times 4 = 20$

5 cakes in each box. 4 boxes equals 20 cakes altogether.

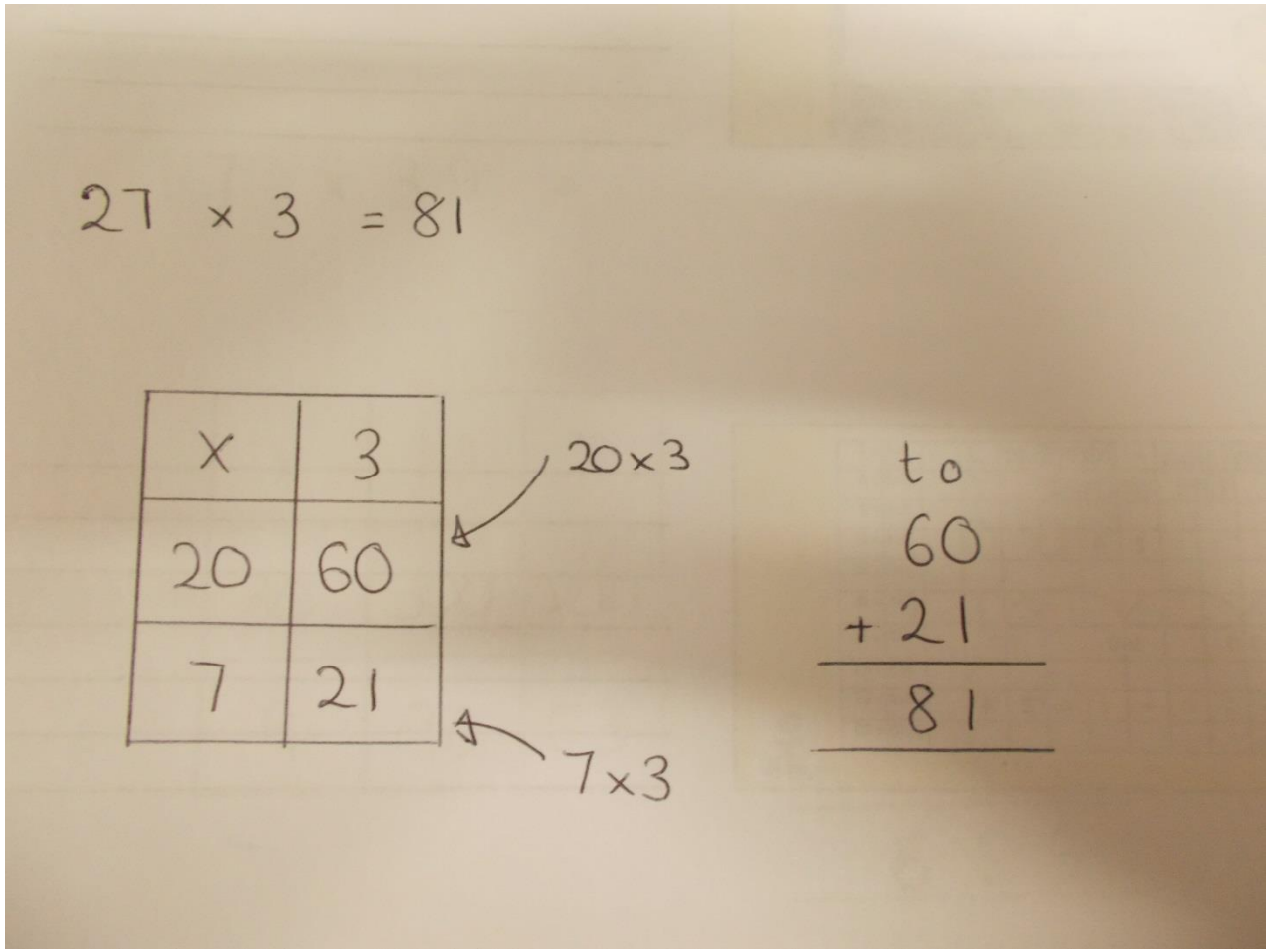


$5 \times 4 = 20$

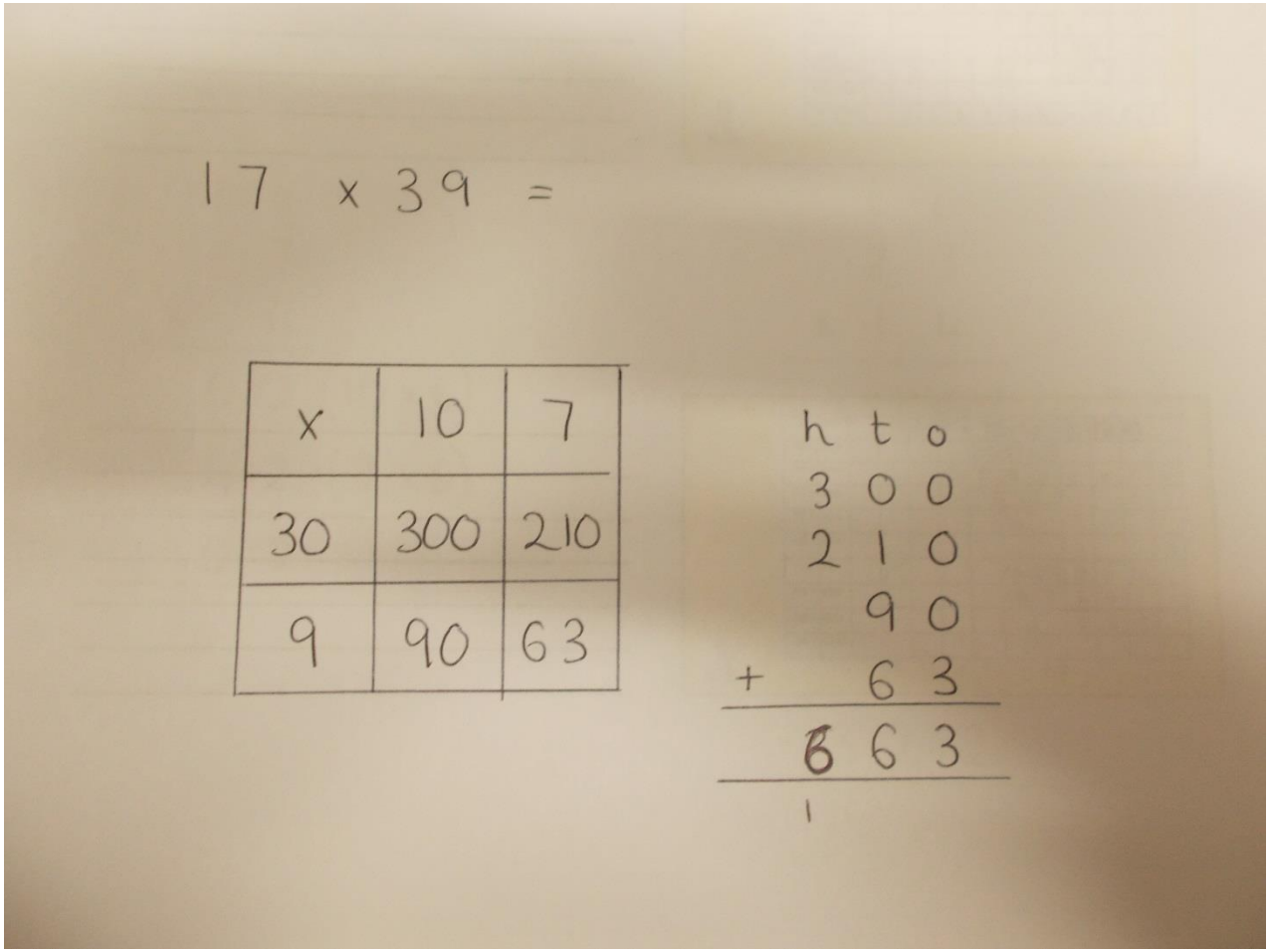
Chunking on a numberline



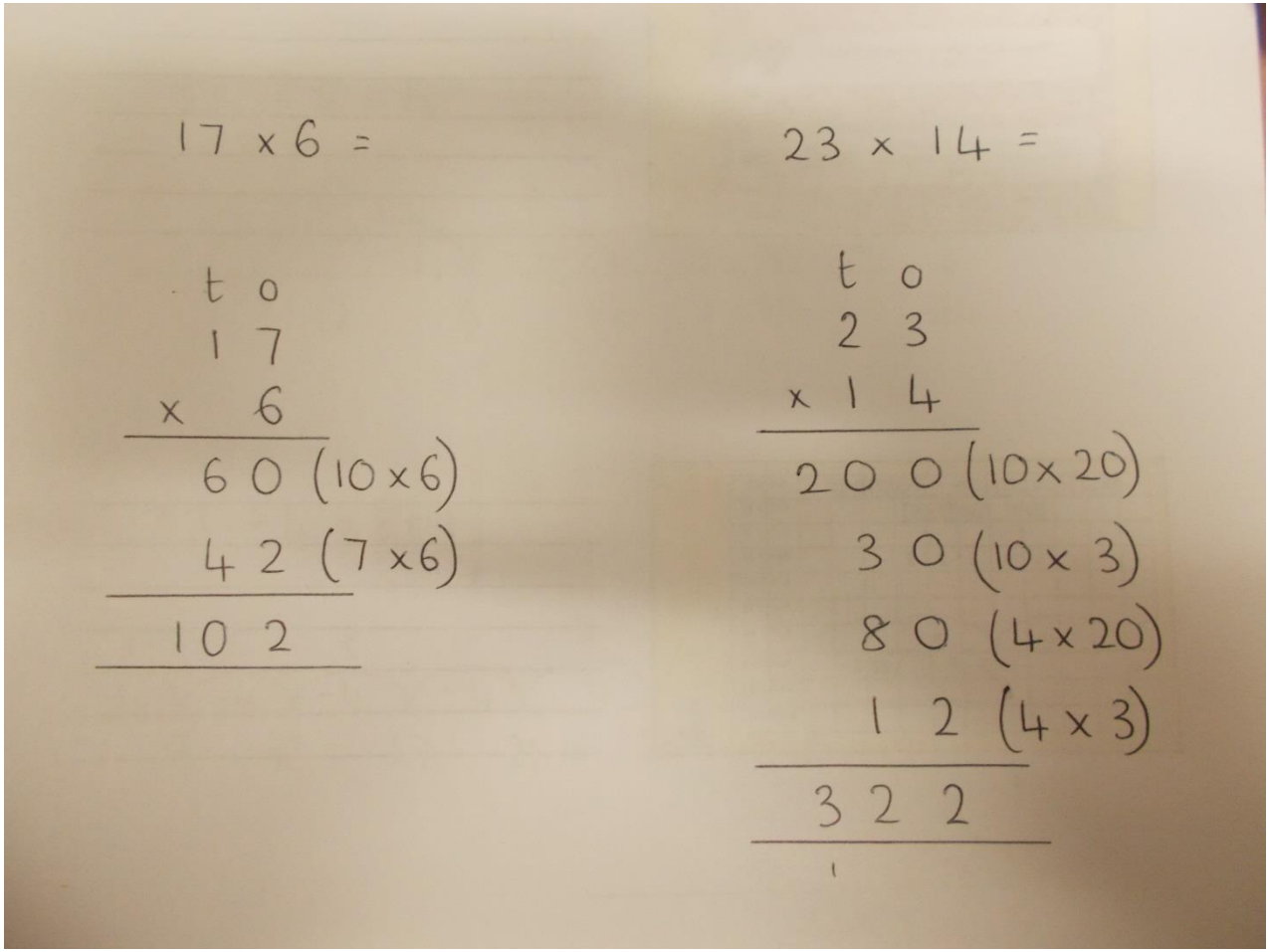
Grid (2digit x 1digit)



Grid (2digit x 2digit)



Expanded column method



Progression in division

- Sharing
- Sharing and grouping
- Jottings to share or group
- Repeated subtraction on a number line
- Chunking on a number line
- Vertical chunking
- Bus stop
- Long division (see Upper KS2 presentation)

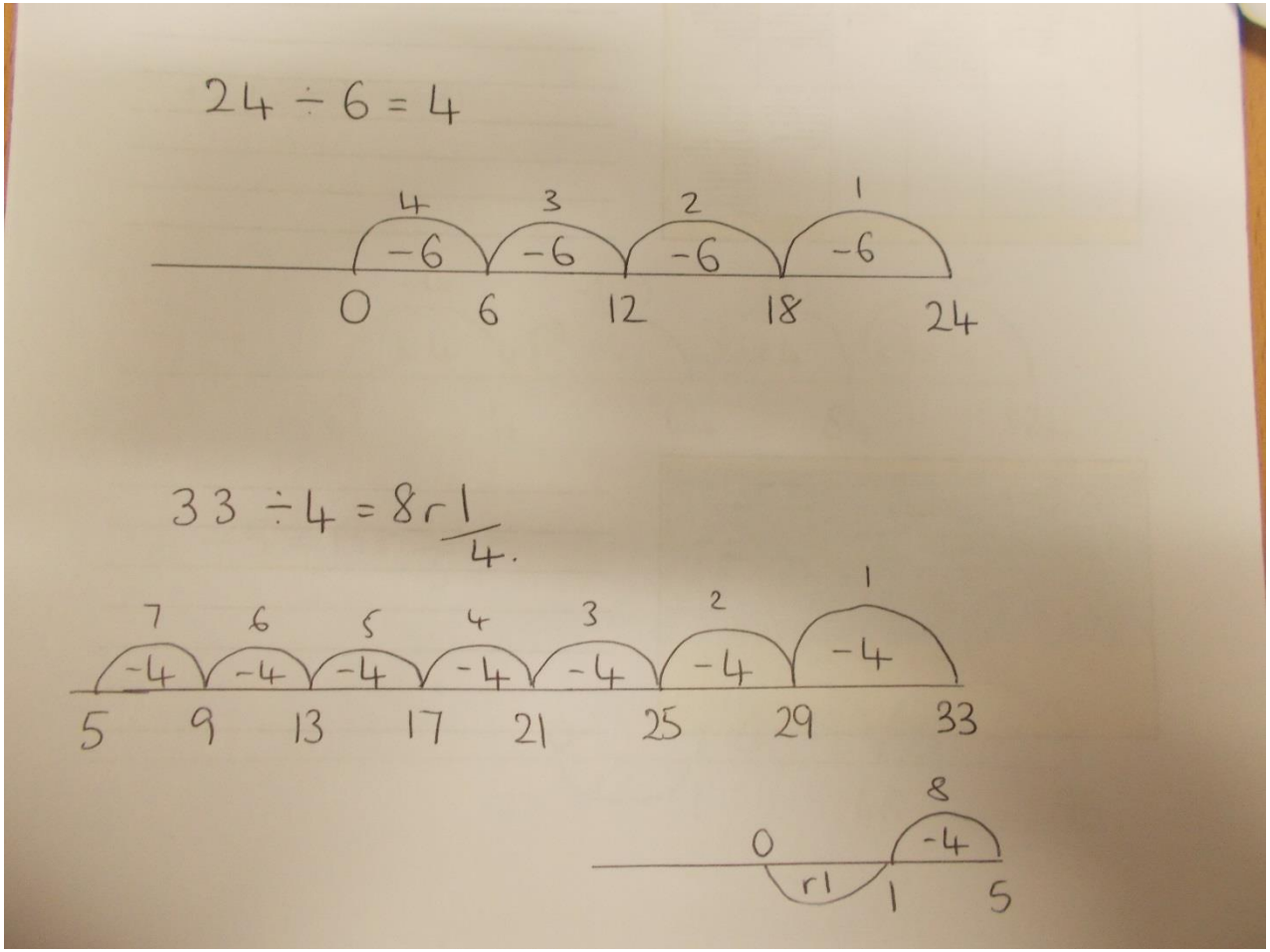
Division in Key Stage 1

Division can be GROUPING or SHARING

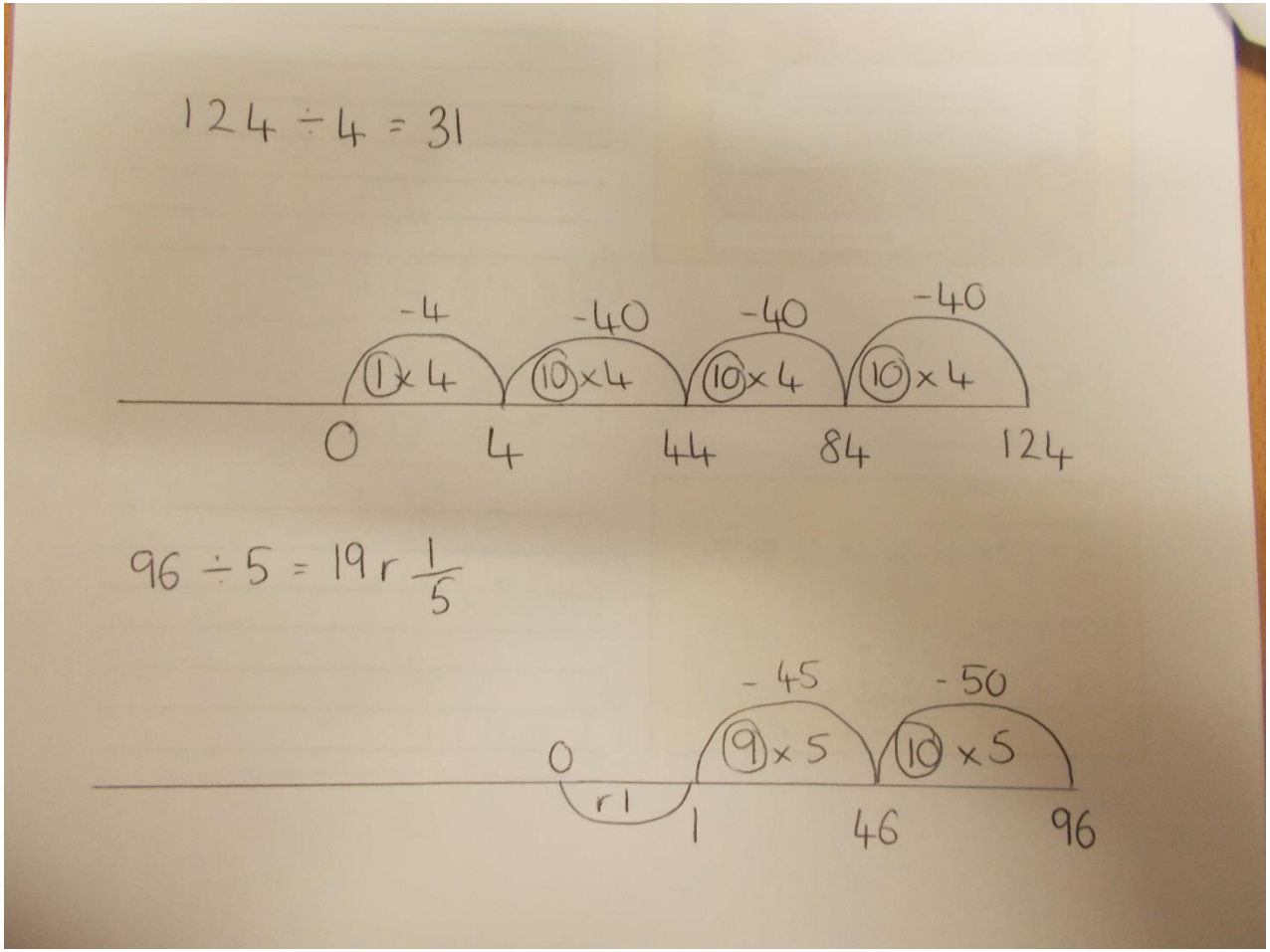
Sharing:
We have 20 marbles in the box to share between 4 children. How many marbles do they get each?
=5 marbles

Grouping:
We have 20 marbles and they come in packets of 4. How many packets were there?
=5 packets

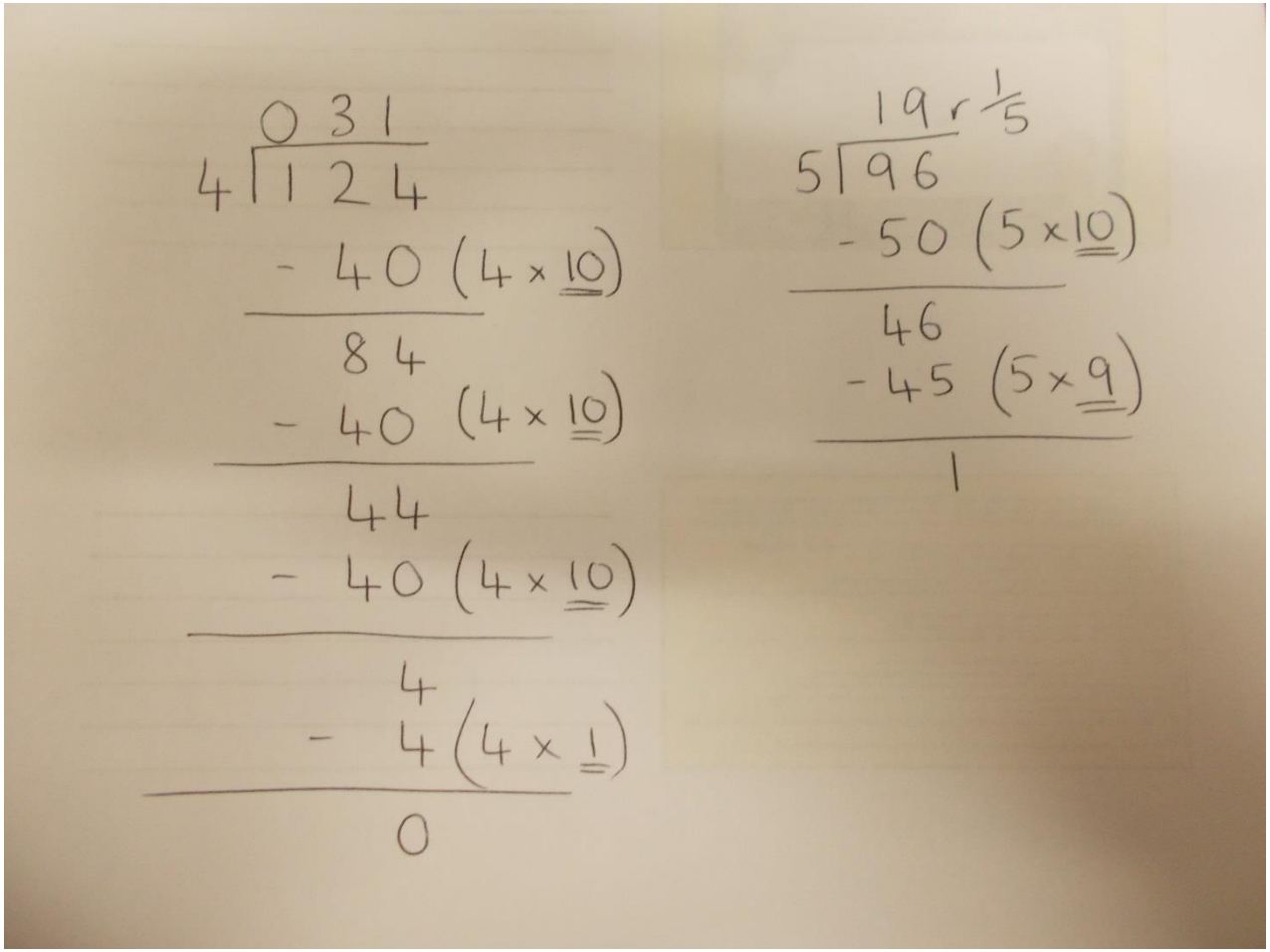
Repeated subtraction



Repeated subtraction – chunking



Vertical chunking and bus stop method



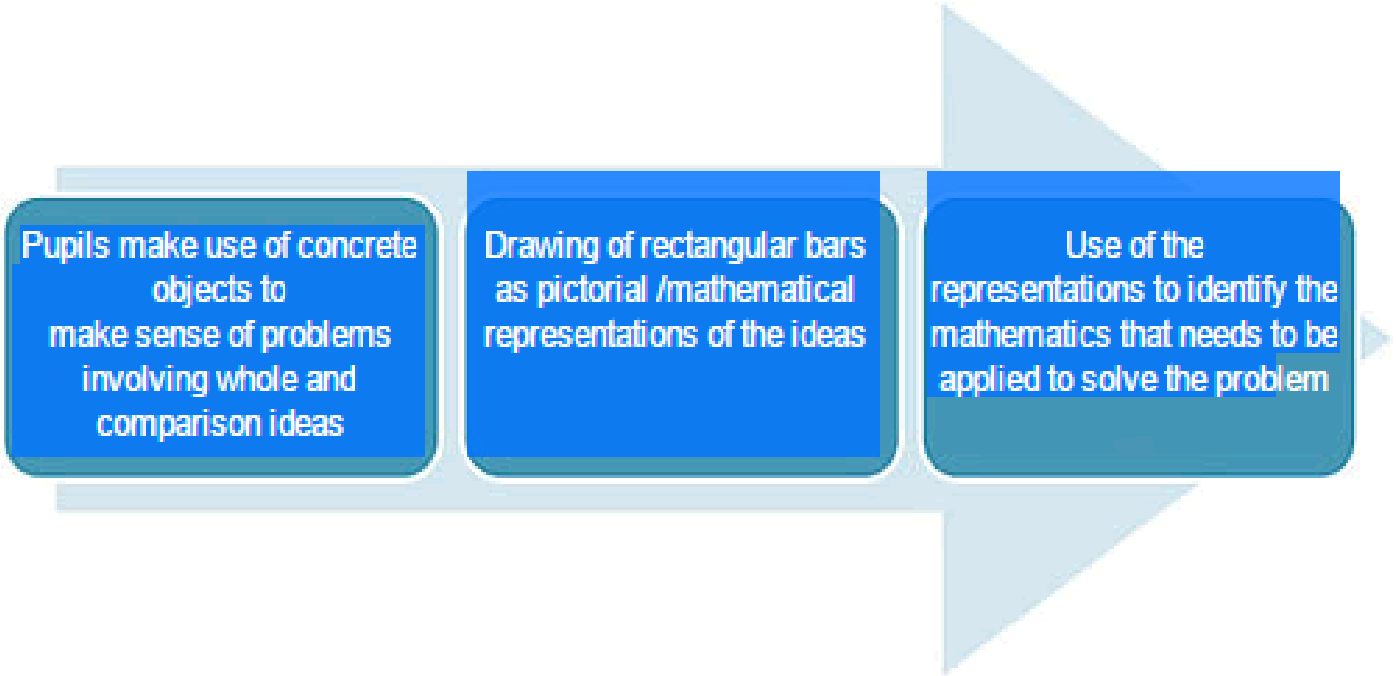
Practical objects CONCRETE

Bar model to represent objects VISUAL

Calculation strategies ABSTRACT

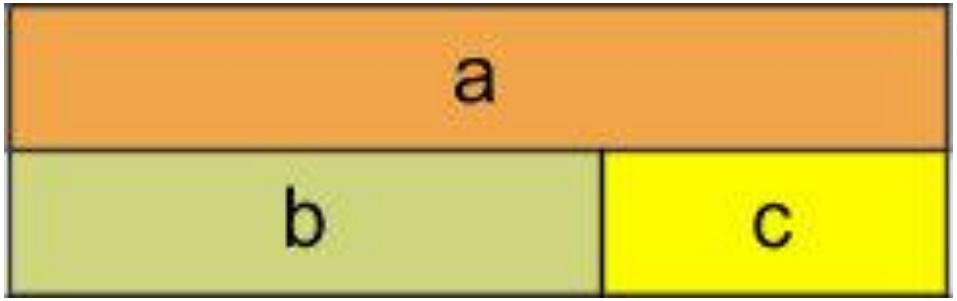
The bar model is used in Singapore and other countries, such as Japan and the USA, to support children in problem solving. It is not a method for solving problems, but a way of revealing the mathematical structure within a problem and gaining insight and clarity as to how to solve it.

It supports the transformation of real life problems into a mathematical form and can bridge the gap between concrete mathematical experiences and abstract representations.



concrete	visual	abstract

Addition and Subtraction



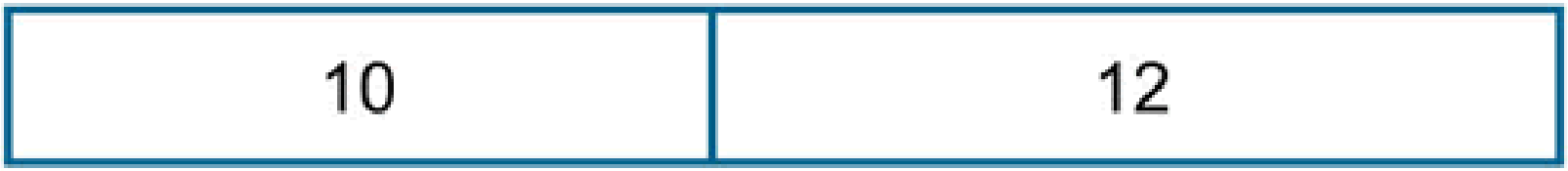
This diagram shows all of the following relationships;
a = b + c ; a = c + b ; a – b = c ; a – c = b

To prepare young children for the bar model it is a good idea to encourage them to line up objects in a linear arrangement when representing addition and subtraction problems.

Such arrangements will also help children to organise their counting. The physical objects can then be replaced, in time, with linking cubes and with a bar drawn next to it. The question can then be asked “what’s the same, what’s different?” to support the children in their reasoning and in making sense of the bar as an abstract representation of the physical objects.

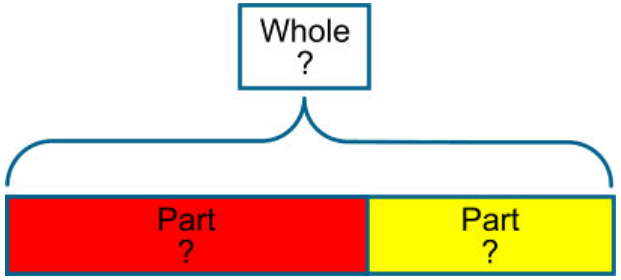
It is useful for children to work in pairs with one manipulating the cubes, while the other records by drawing the bars and then writing the number sentence underneath. The children can then swap roles.

Sam had 10 red marbles and 12 blue marbles. How many marbles did he have altogether?



$10 + 12 = 22$

In problems involving addition and subtraction there are three possible unknowns as illustrated below and given the value of two of them the third can be found.



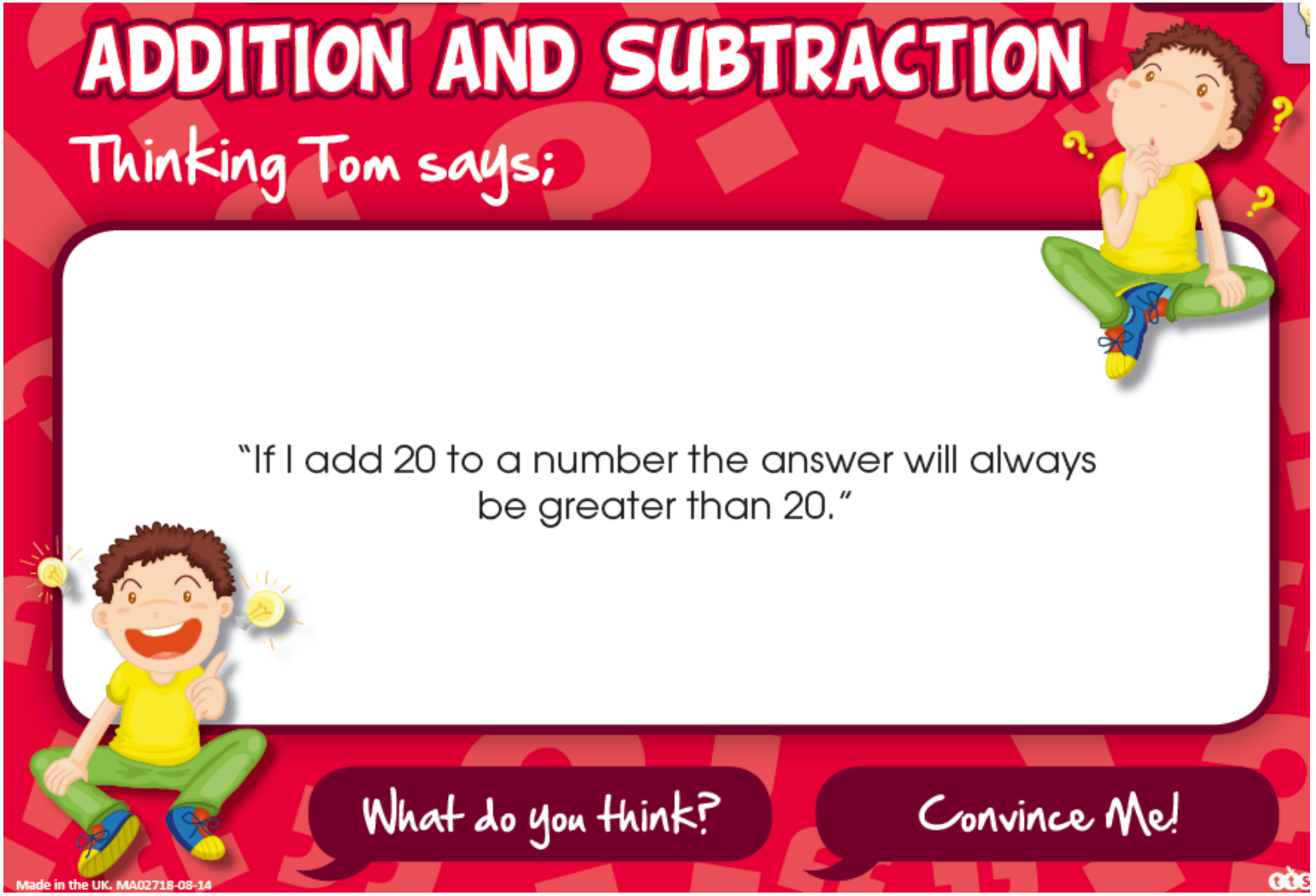
Why are images important?

- Resources, models and images help children to visualise and understand mathematical concepts
- They build up – and remember – the mental picture in their minds
- They should be available throughout the primary years. Children will rely on them less and less

When tackling a mathematical problem, children will ask themselves…

- What do I know about these numbers?
- Can I do this in my head?
- Do I know the approximate size of the answer?
- If I can't do it all in my head, what do I need to write down to help me?

Thinking outside of the box…



Times tables

Order of learning multiplication facts:

10x 5x 2x 4x 8x 3x 6x 9x 12x 7x x11

And related division facts:

56	
8	7

8 X 7 = 56
7 x 8 = 56
56 ÷ 7 = 8
56 ÷ 8 = 7

What else can we work out?

Written vs Mental Strategies
(Arithmetic Paper)

Mental calculations

7 + 6 = 17 + 6 = _ + _ + _ = 14
73 + 40 = 98 – 18 =
597 + 7 = 804 – 70 =
6.7 + 0.4 = 19354 – 500 =
6.32 + __ = 8

Children need to be able to decide when to do a calculation mentally or using a written method.

1003 – 7 =

£3.99 x 5 =

A useful checklist for what to look out for when assessing a pupil's understanding might be:

A pupil really understands a mathematical concept, idea or technique if he or she can:

- **describe** it in his or her own words;
- **represent** it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)
- **explain** it to someone else;
- **make up his or her own examples** (and non-examples) of it;
- **see connections** between it and other facts or ideas;
- **recognise it in new situations and contexts**;
- **make use of it in various ways**, including in new situations.

<u>Why does this work?</u> *Convince me.*

Use reasoning prompts to do this:

- Why do you think that …?
- Can you explain why that is right?
- How do you know?
- How did you reach that conclusion?
- What might explain that …?
- How is that possible?
- Can you show me …?
- Is there another way …?
- What explanation do you think is best …?
- Have you tried all the possible cases?
- Does it always work? Why?
- What do you notice when …?

"Above all – have fun! Children often say that maths is their favourite subject and they get quicker and better at it when they understand what they are doing. There's no need for you to "teach" your child: you help by helping them to explain their thinking and their understanding"

From "Count on me – 200 ways to help with mathematics"

- “It’s the school’s job to provide the structured learning….Your role is to nurture and support your child’s mathematical knowledge away from school, to bring it into their real lives and, most important of all, to turn it into an exciting adventure”

From “Maths for Mums and Dads” – Rob Eastaway and Mike Askew

ISBN: 978-0-224-08635-6

Have your questions been answered?